Connecting de Bruijn Graphs

Giulia Bernadini, Huiping Chen, Inge Li Gørtz, ***Christoffer Krogh***, Grigorios Loukides, Solon P. Pissis, Leen Stougie, Michelle Sweering

Overview

- Previous Work
- This Work

Previous Work

Making de Bruijn Graphs Eulerian

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- Part of: Volume: 33rd Annual Symposium on Combinatorial Pattern Matching (CPM 2022)
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Making de Bruijn Graphs Eulerian

de Bruijn Graphs

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- Collection of length k strings

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- Vertices are length $k-1$ subtrings

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- Edges iff corresponding string exists in collection

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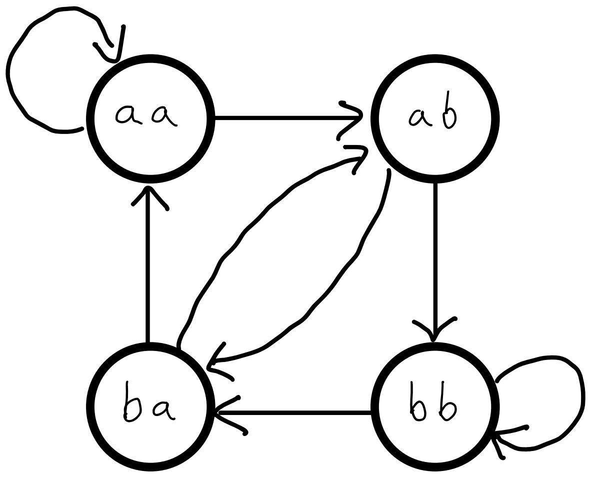
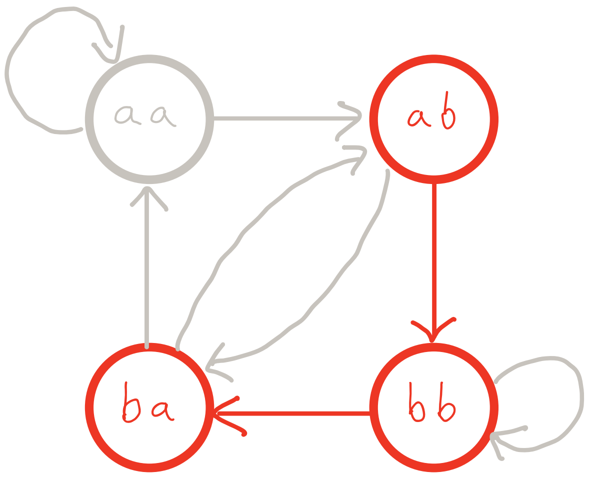


$\Sigma = \{a, b\}$
$k = 3$



$\{abb, bba\}$
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Eulerian Graphs

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- Circuit of every edge exactly once

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- Euler’s Theorem:
    1. Edges must be connected
    2. Vertices must be balanced

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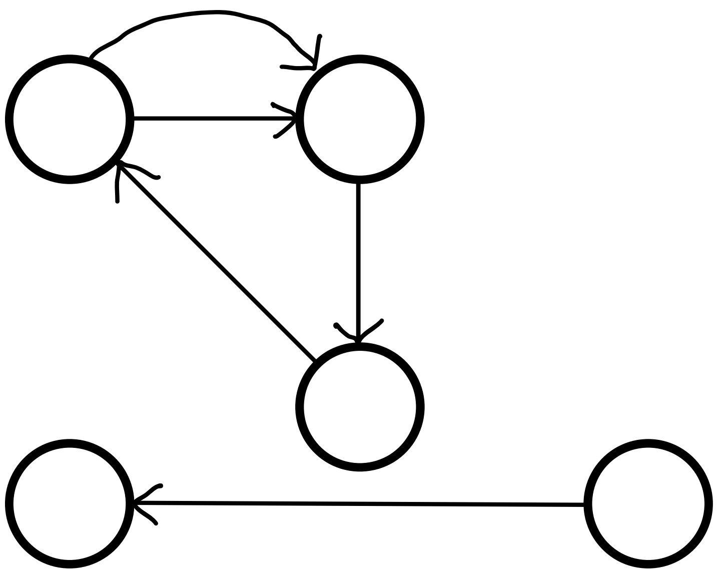

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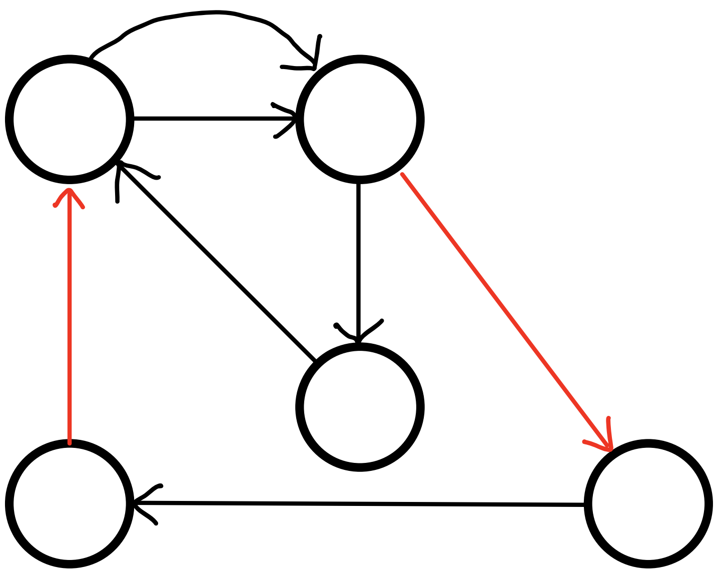

Main Problem

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Making de Bruijn Graphs Eulerian

Eulerian Extension of de Bruijn Graphs (EXTEND-DBG)

Main Problem

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Making de Bruijn Graphs Eulerian

Eulerian Extension of de Bruijn Graphs (EXTEND-DBG)

- Given a de Bruijn graph

Main Problem

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Eulerian Extension of de Bruijn Graphs (EXTEND-DBG)

- Given a de Bruijn graph
- Make Eulerian from the complete de Bruijn graph

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Making de Bruijn Graphs Eulerian

Eulerian Extension of de Bruijn Graphs (EXTEND-DBG)

- Given a de Bruijn graph
- Make Eulerian from the complete de Bruijn graph
- Minimize number of new edges

Motivation

DNA sequencing

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- $caaacgca \Rightarrow \{caa, aaa, aac, acg, cgc, gca\}$

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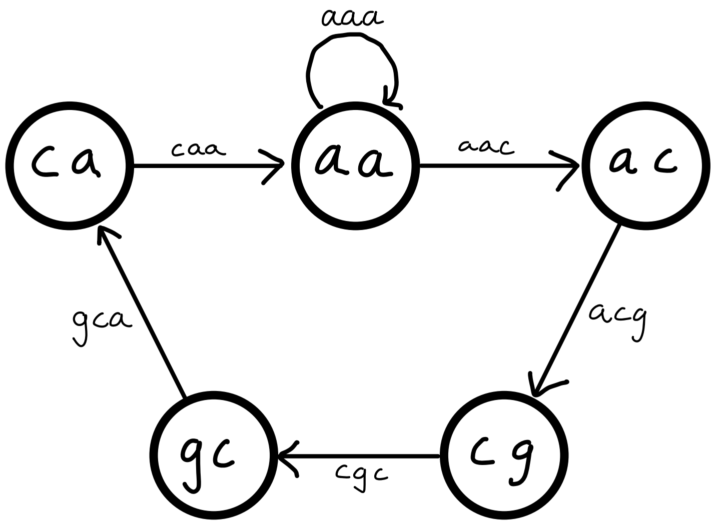

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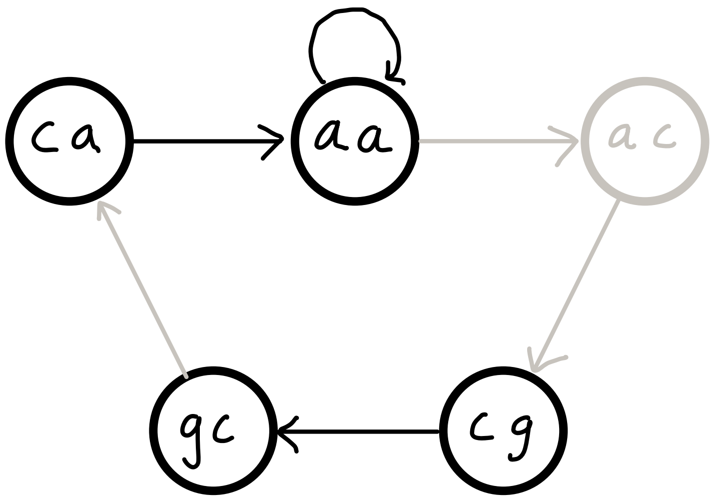

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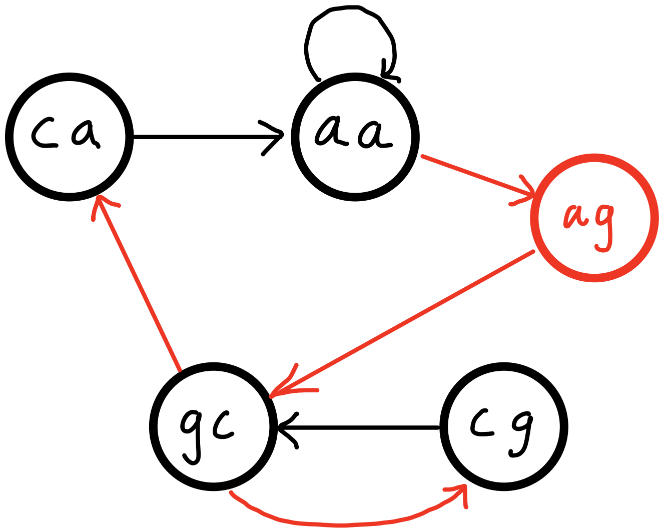

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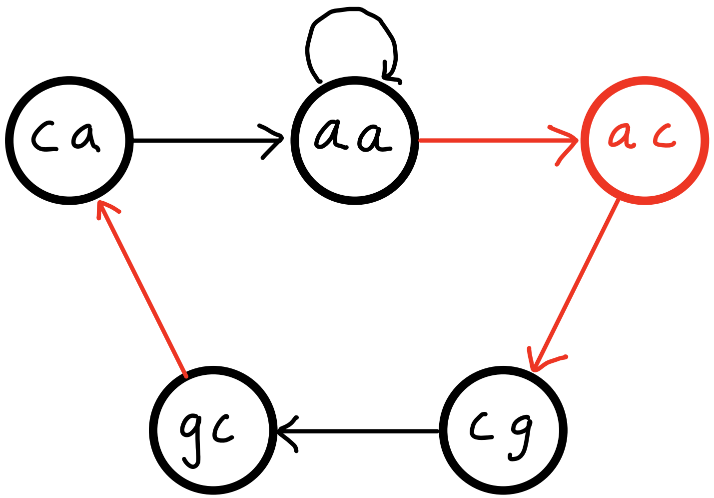

Hardness

- EXTEND-DBG is NP-hard
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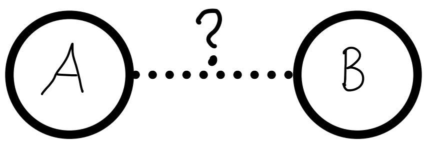
- Split problem in two:
  1. Connect de Bruijn Graph
  2. Balance de Bruijn Graph

Connect de Bruijn Graph

$$A = A_L \, X \, A_R$$

$$B = B_L \, X \, B_R$$



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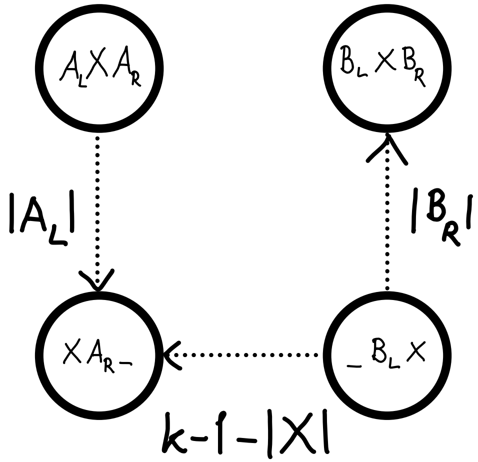

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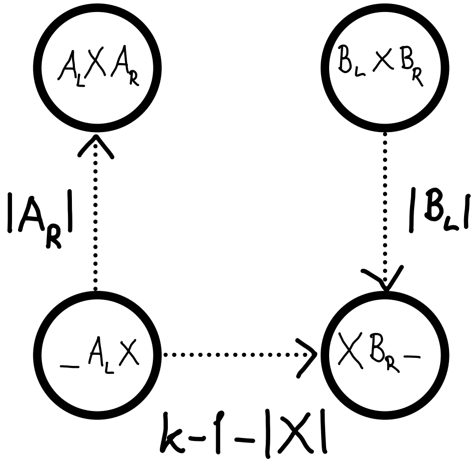

Connect de Bruijn Graph

$A = A_L \, X \, A_R$

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$$d(A,B) = k - 1 - |X| + \min\{A_L + B_R, A_R + B_L\}$$

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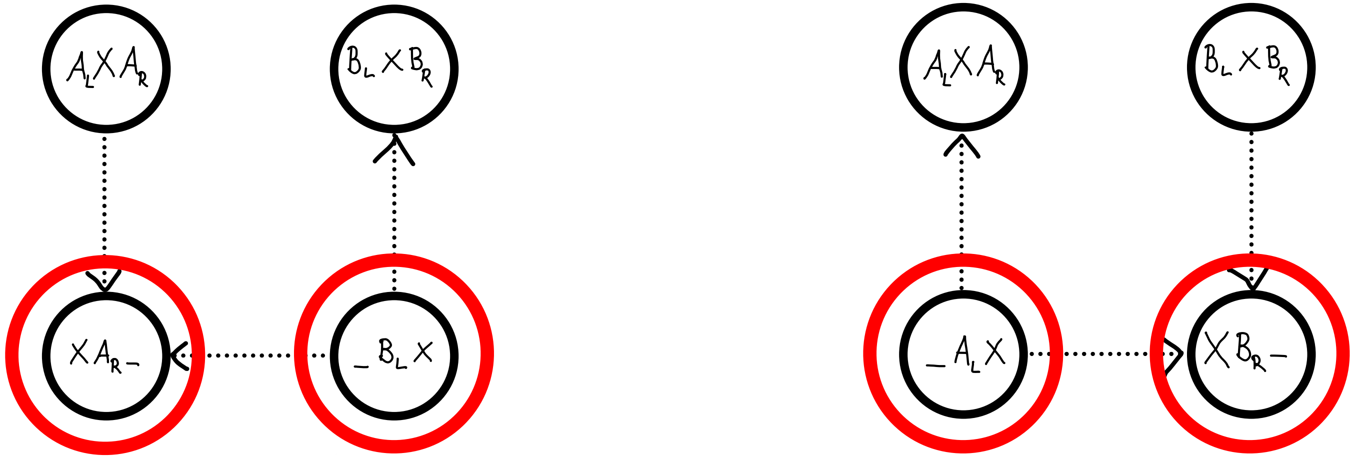

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Connect de Bruijn Graph with Paths

CONNECT-DBG-P

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- Given a de Bruijn Graph

Connect de Bruijn Graph with Paths

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- Given a de Bruijn Graph
- Weakly connect adding only directed paths

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Solve in $\mathcal{O}(|V| k \log d + |E|)$ time,

d is the number of connected components

Connect de Bruijn Graph with Paths

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Solve in $\mathcal{O}(|V|k\log d + |E|)$ time,

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Balance de Bruijn Graph

BALANCE-DBG

- Given a de Bruijn Graph
- **Balance all vertices**
- Minimize number of new edges

Connect de Bruijn Graph with Paths

CONNECT-DBG-P

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Solve in $\mathcal{O}(|V|k \log d + |E|)$ time,

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Balance de Bruijn Graph

BALANCE-DBG

- Given a de Bruijn Graph
- Balance all vertices
- Minimize number of new edges

Solve in $\mathcal{O}(k|V| + |E| + |A|)$ time,

$|A|$ is the number of added edges

Overview

- Previous Work
- This Work

DTU

This Work

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Results

1. Connecting de Bruijn Graphs is NP-hard

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Connecting de Bruijn Graphs

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1. Connecting de Bruijn Graphs is NP-hard

2. 2-approximation for CONNECT-DBG

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1. Connecting de Bruijn Graphs is NP-hard
2. 2-approximation for CONNECT-DBG*
3. Improved and simplified solution to CONNECT-DBG-P

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Connecting de Bruijn Graphs

Results

1. Connecting de Bruijn Graphs is NP-hard
2. 2-approximation for Connect-DBG*
3. Improved and simplified solution to Connect-DBG-P
4. Integer linear program formulation

Hardness of CONNECT-DBG

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Reduction from Vertex Cover

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Reduction from

Vertex Cover

- Given an undirected graph

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$$\mathcal{I}_{VC} = G(V,E) \quad \Rightarrow \quad \mathcal{I}_{C-dBG}$$

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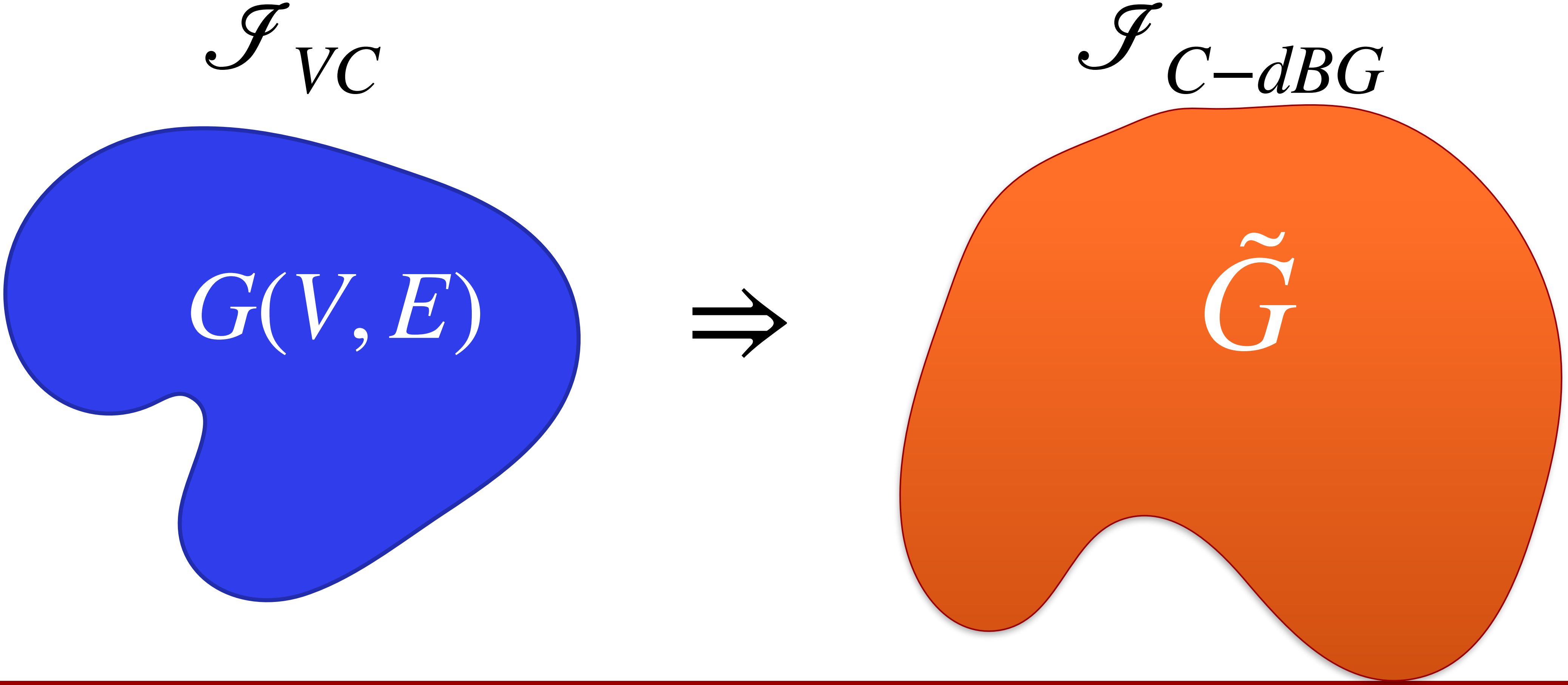

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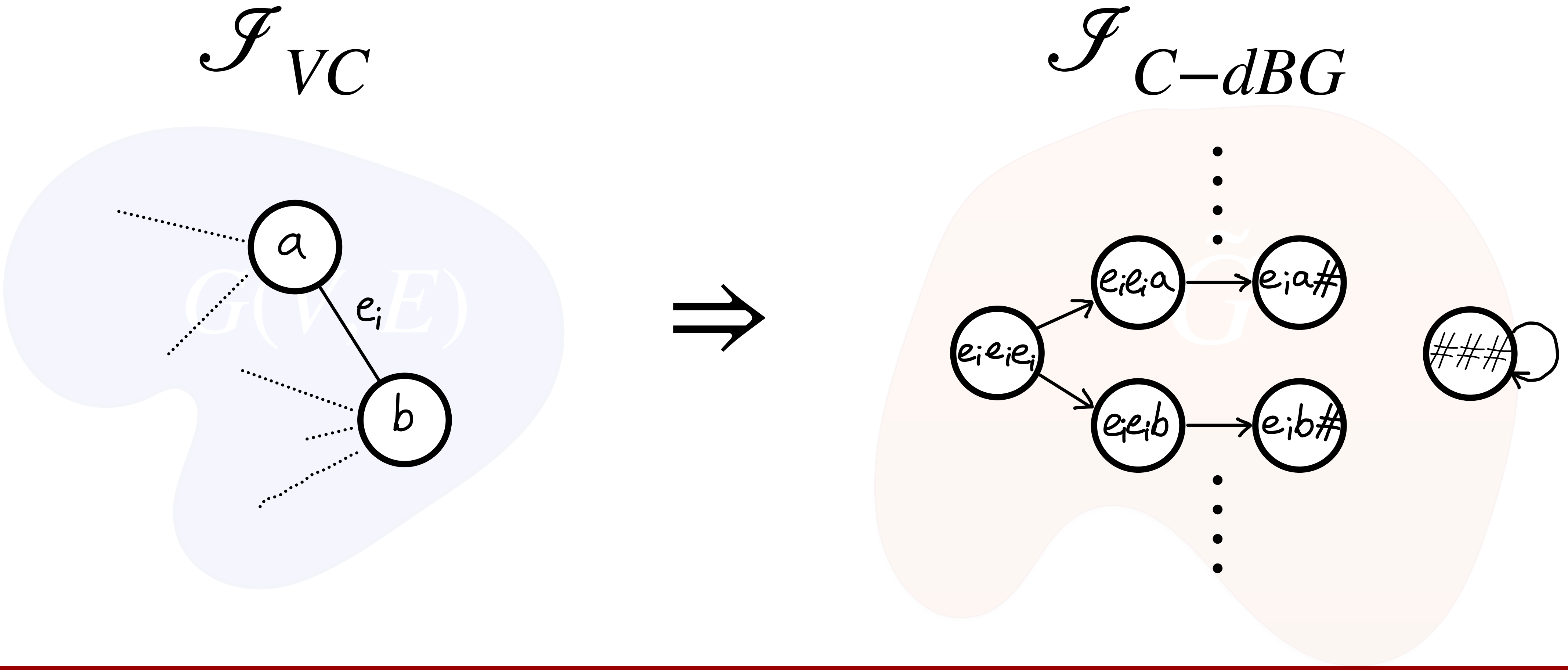

Hardness of Connect-DBG

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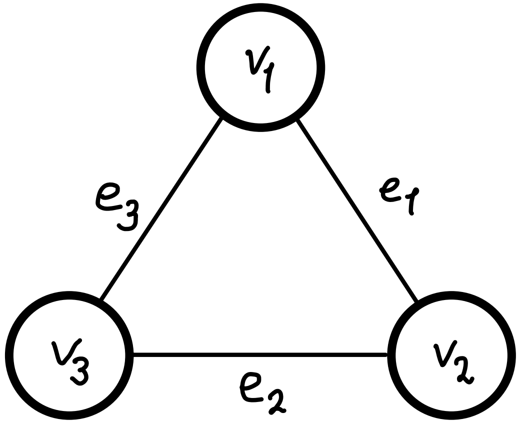

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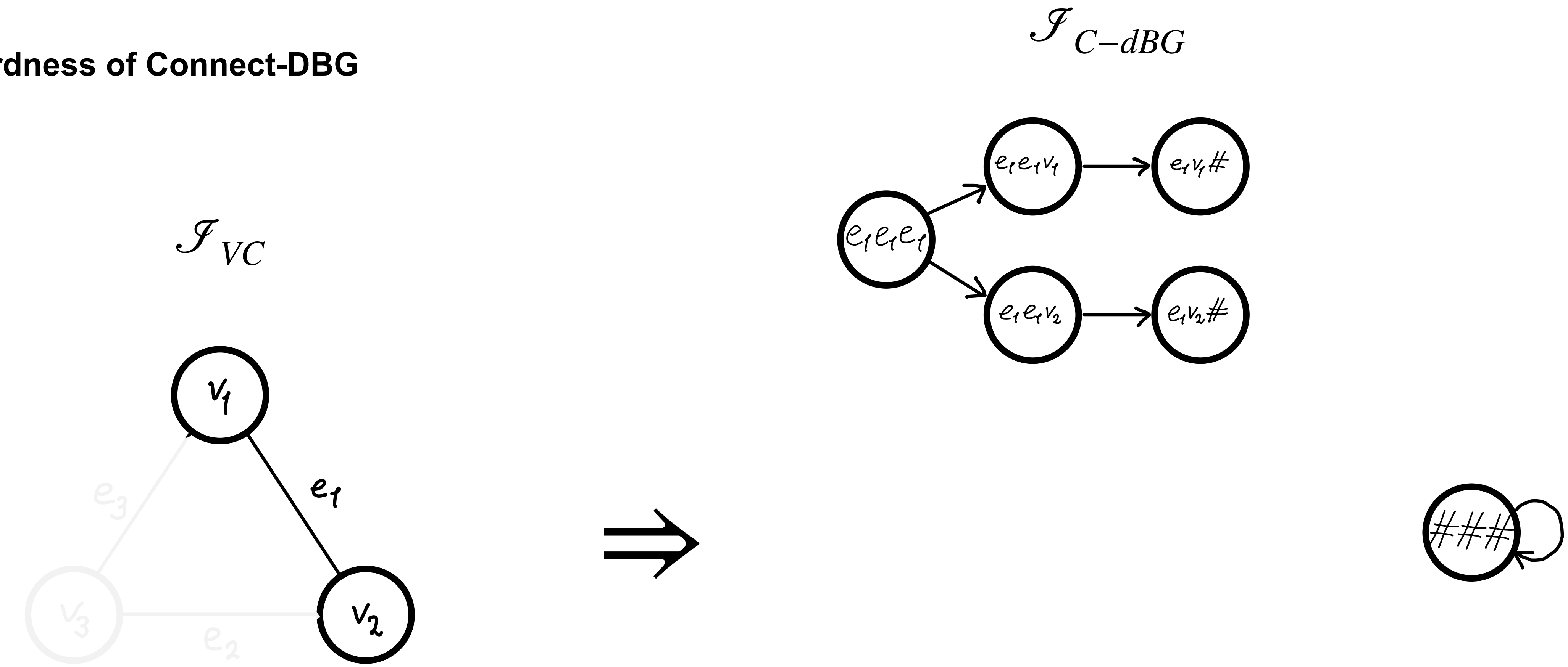

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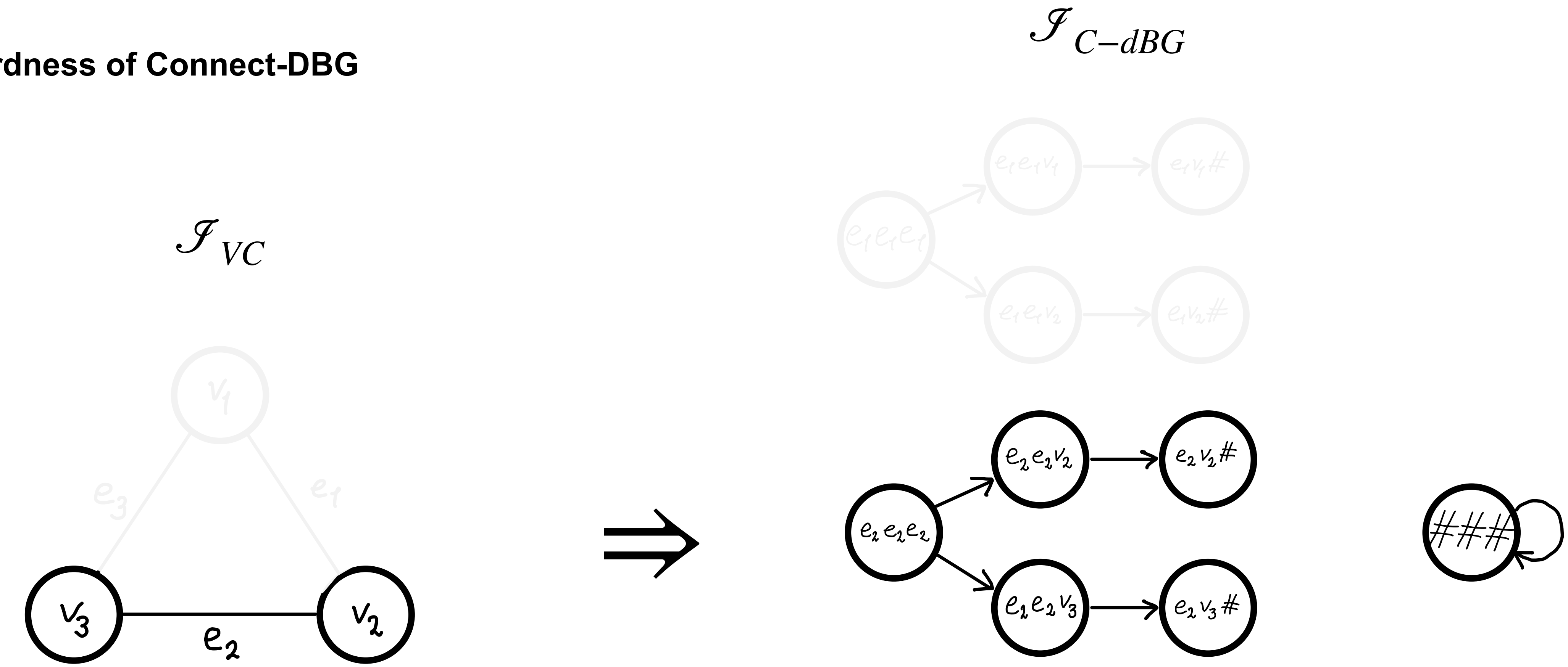

Hardness of Connect-DBG

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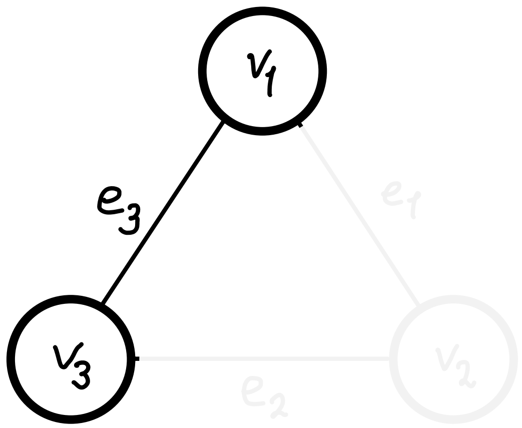
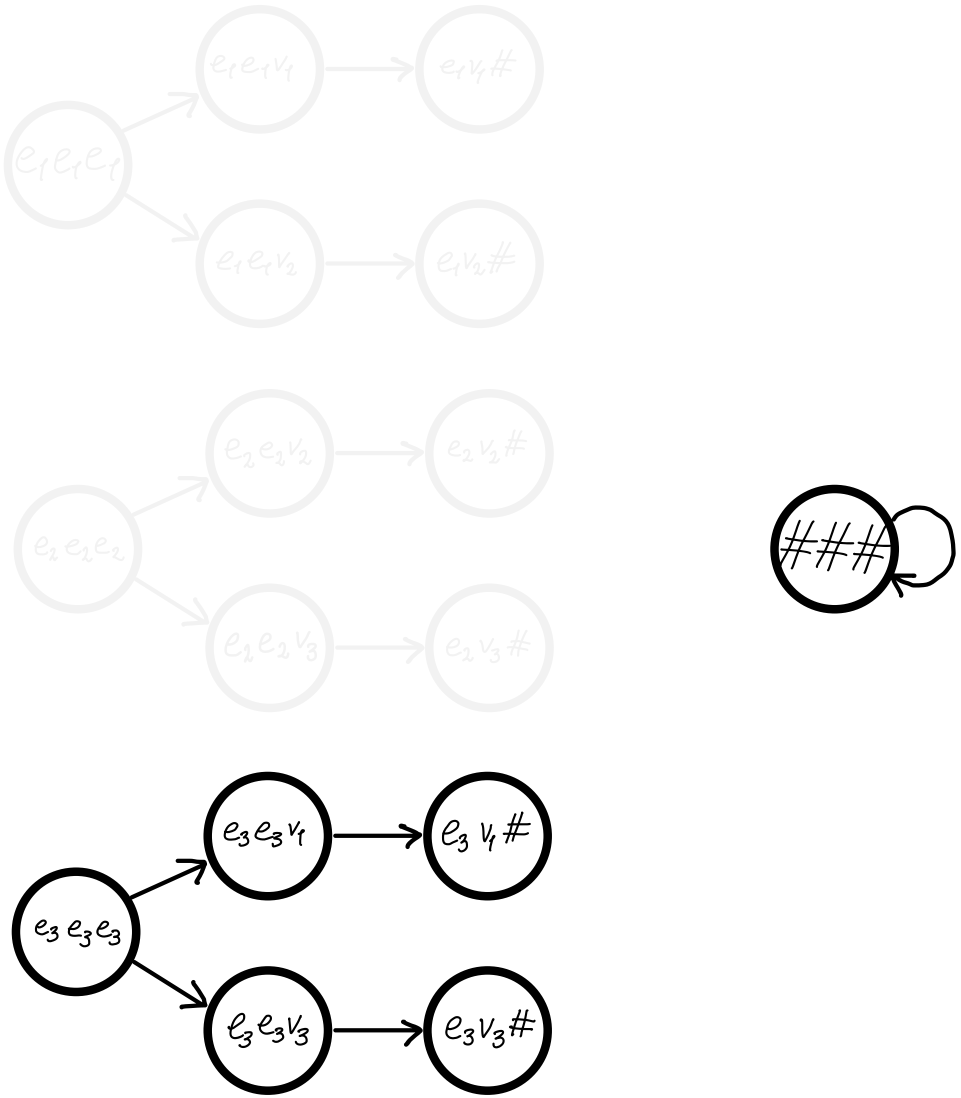


$\Rightarrow$

$\mathcal{I}_{C-dBG}$



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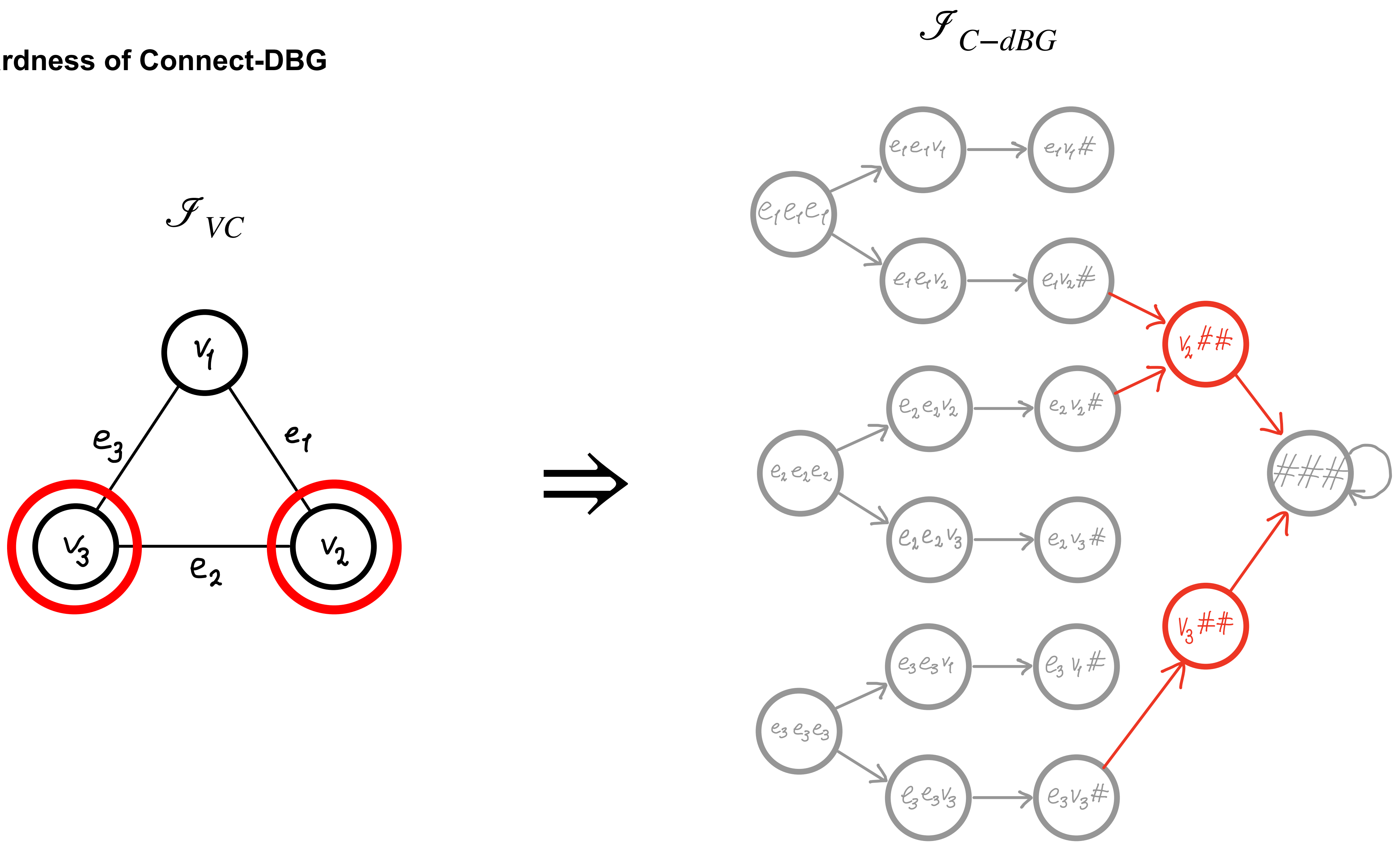
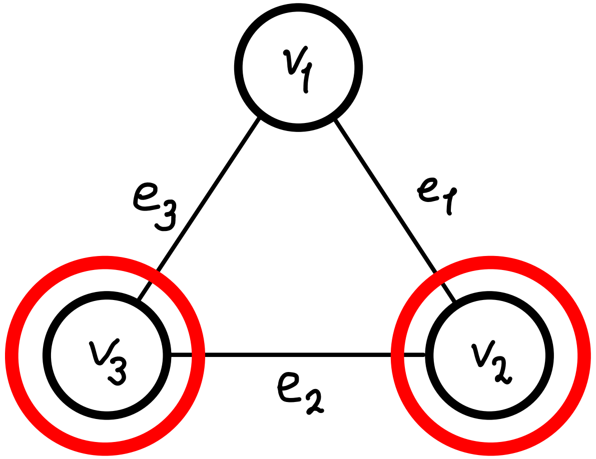


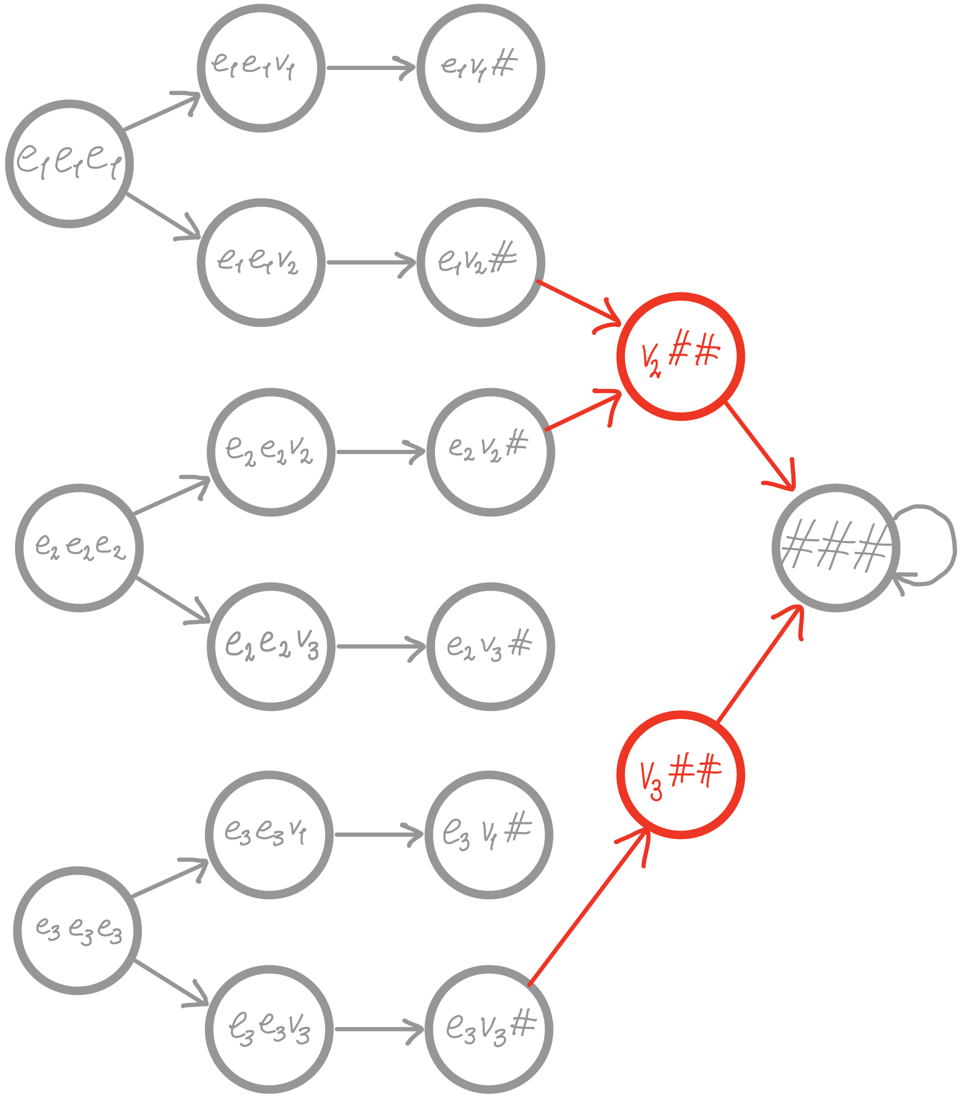
Hardness of Connect-DBG

$$OPT(\mathcal{I}_{VC}) = 2$$



$\Rightarrow$

$$OPT(\mathcal{I}_{C-dBG}) = 2 + |E| = 5$$



Hardness of Connect-DBG

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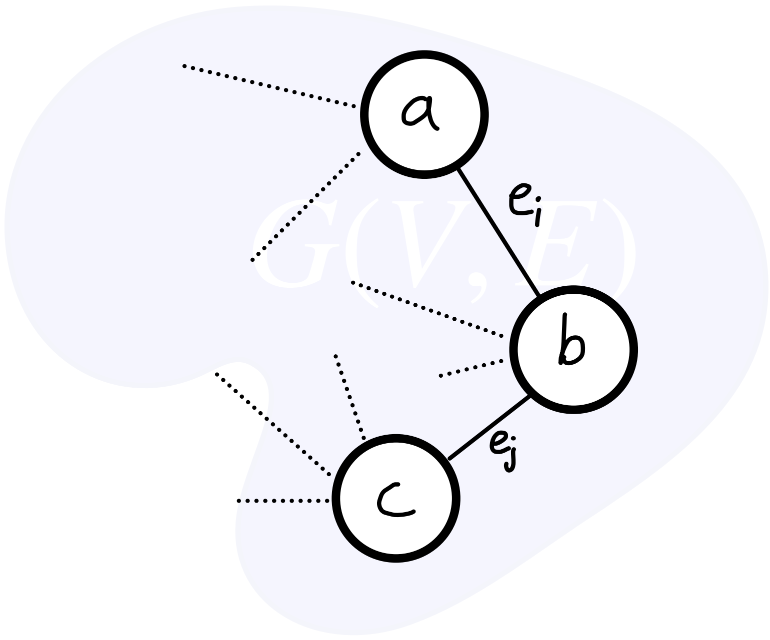
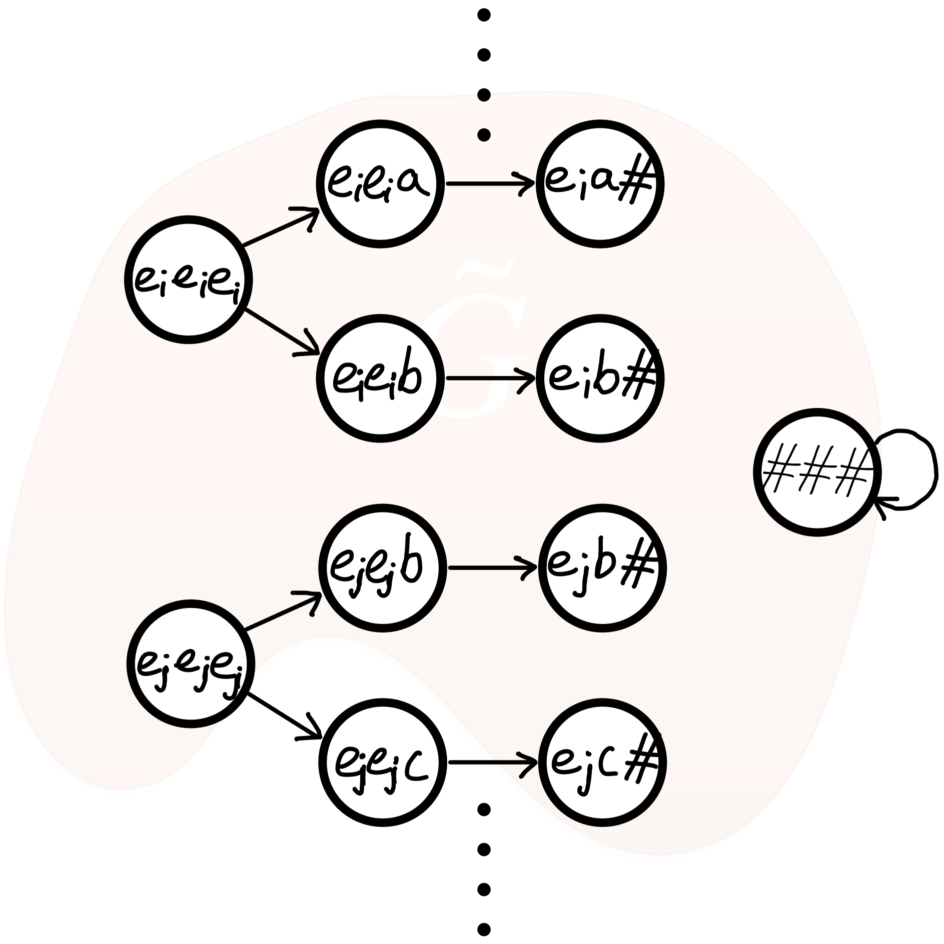


$\Leftarrow$

$\mathcal{I}_{C-dBG}$



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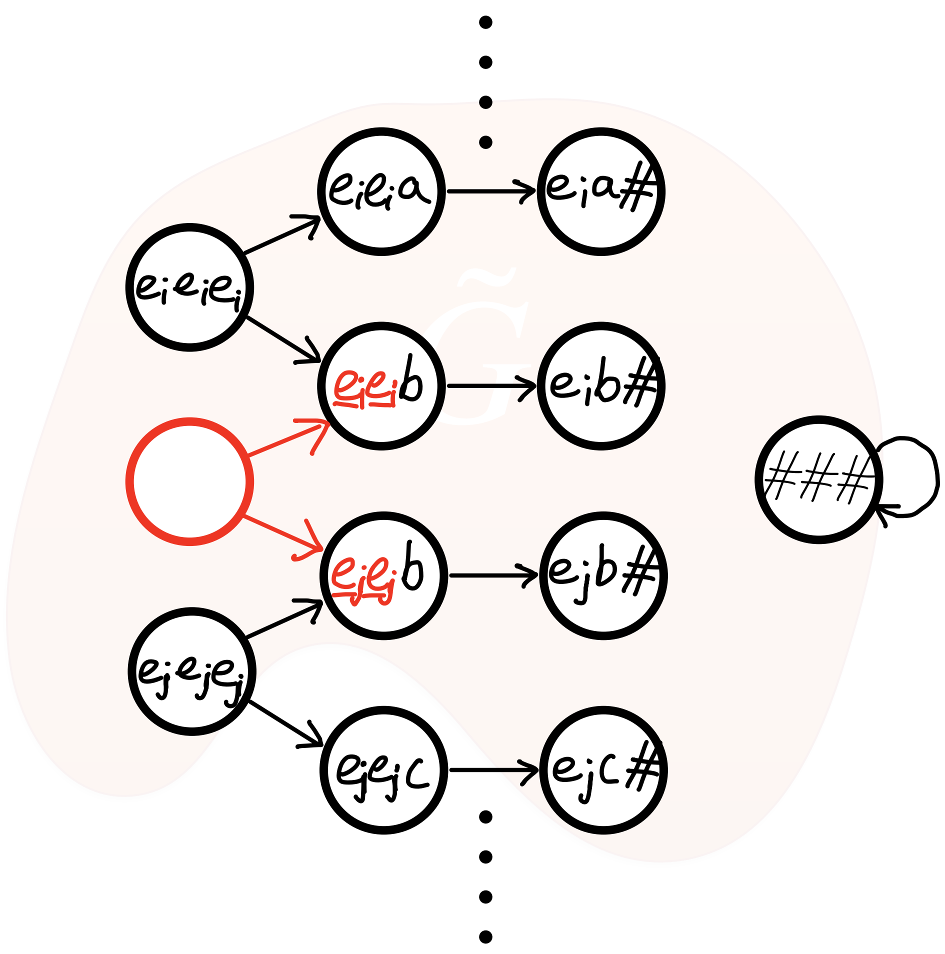

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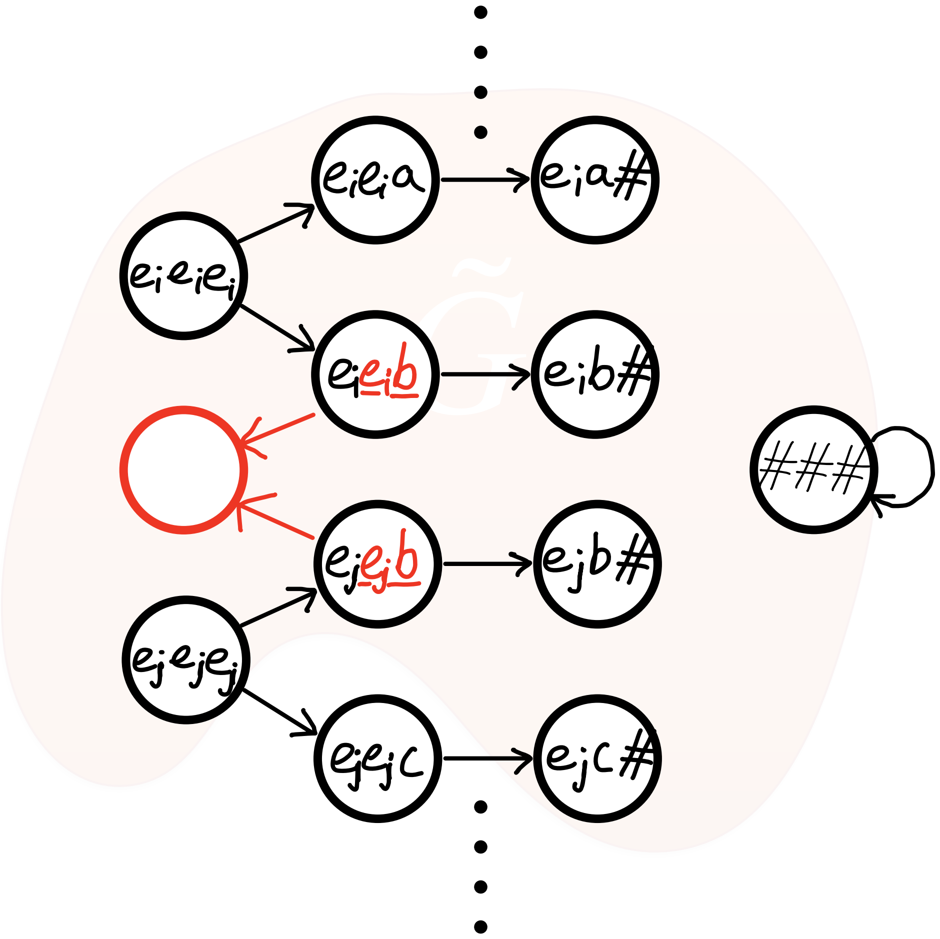

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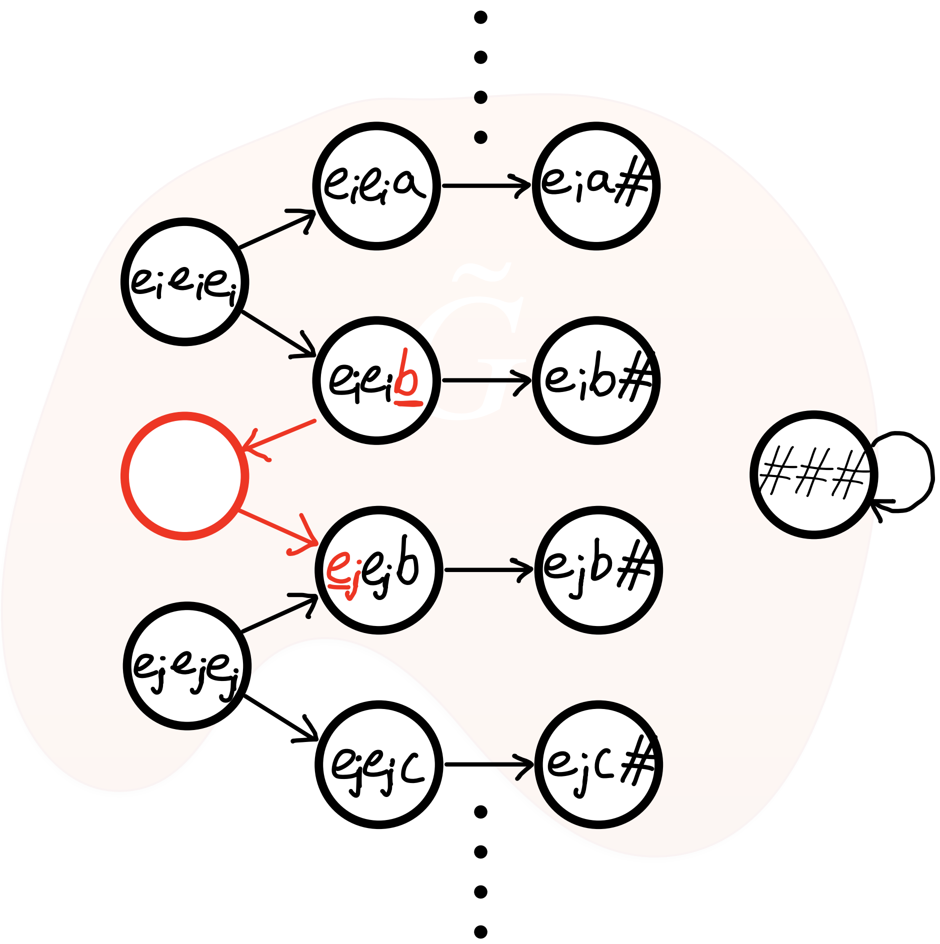

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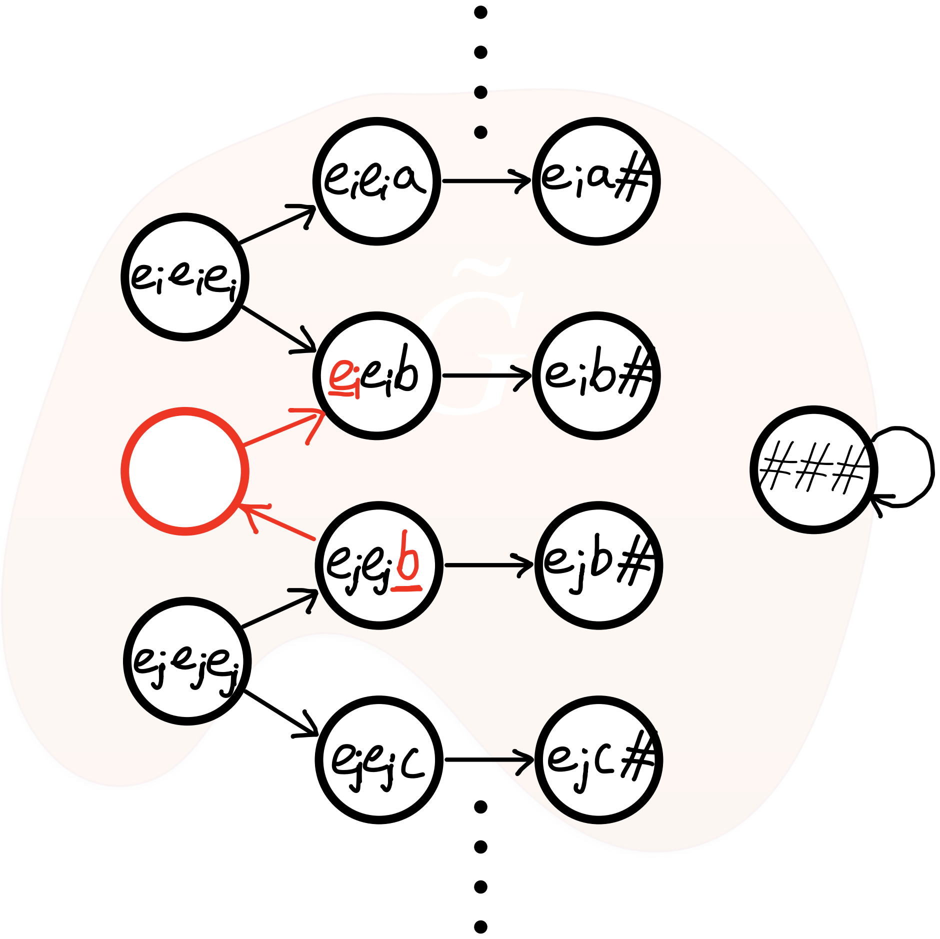

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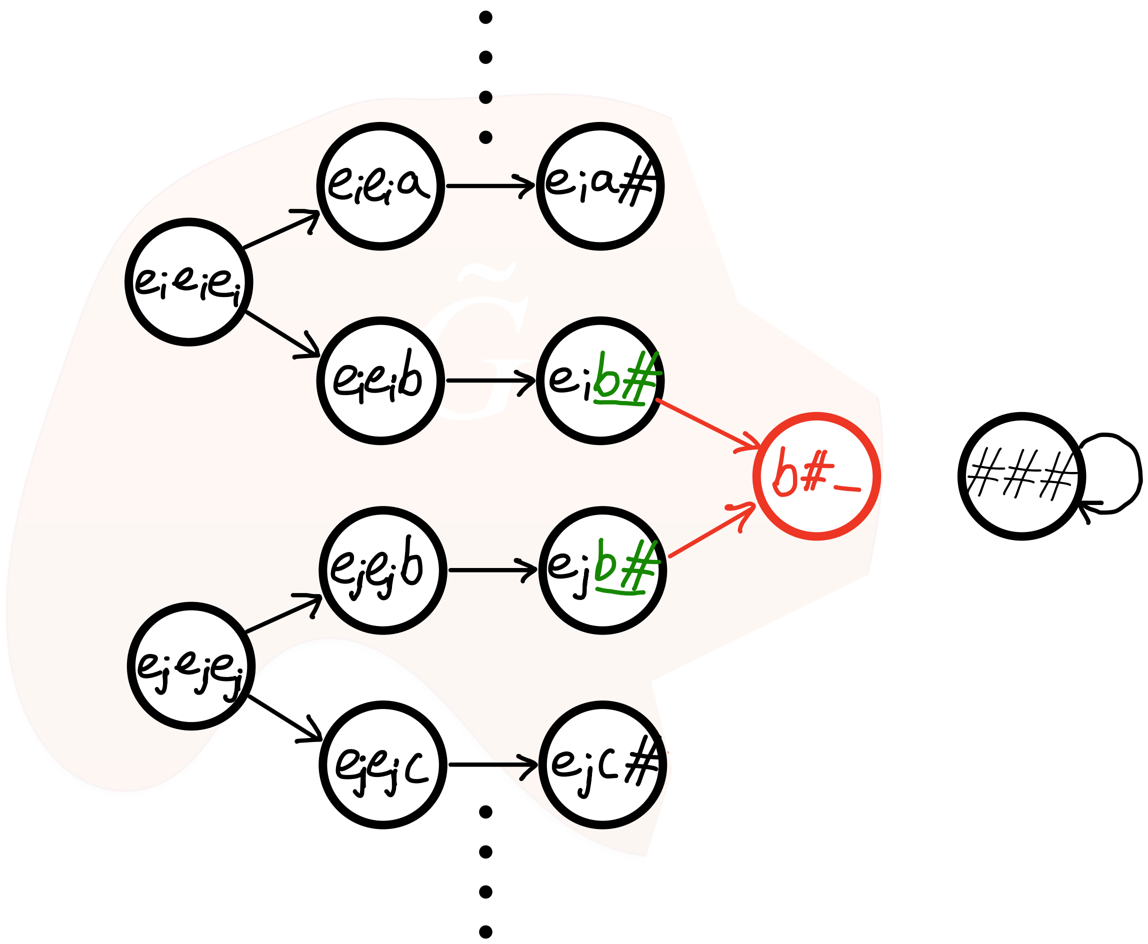

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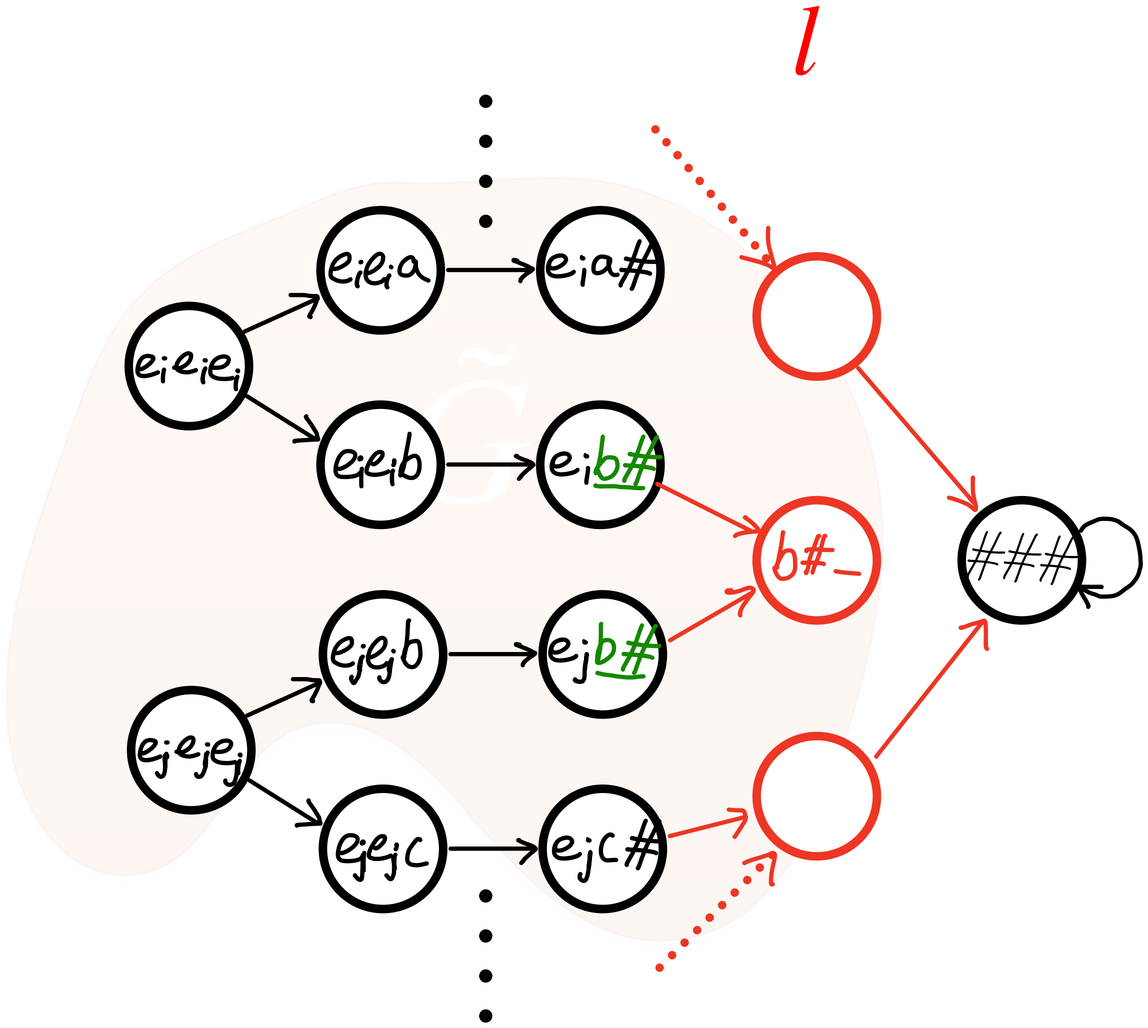
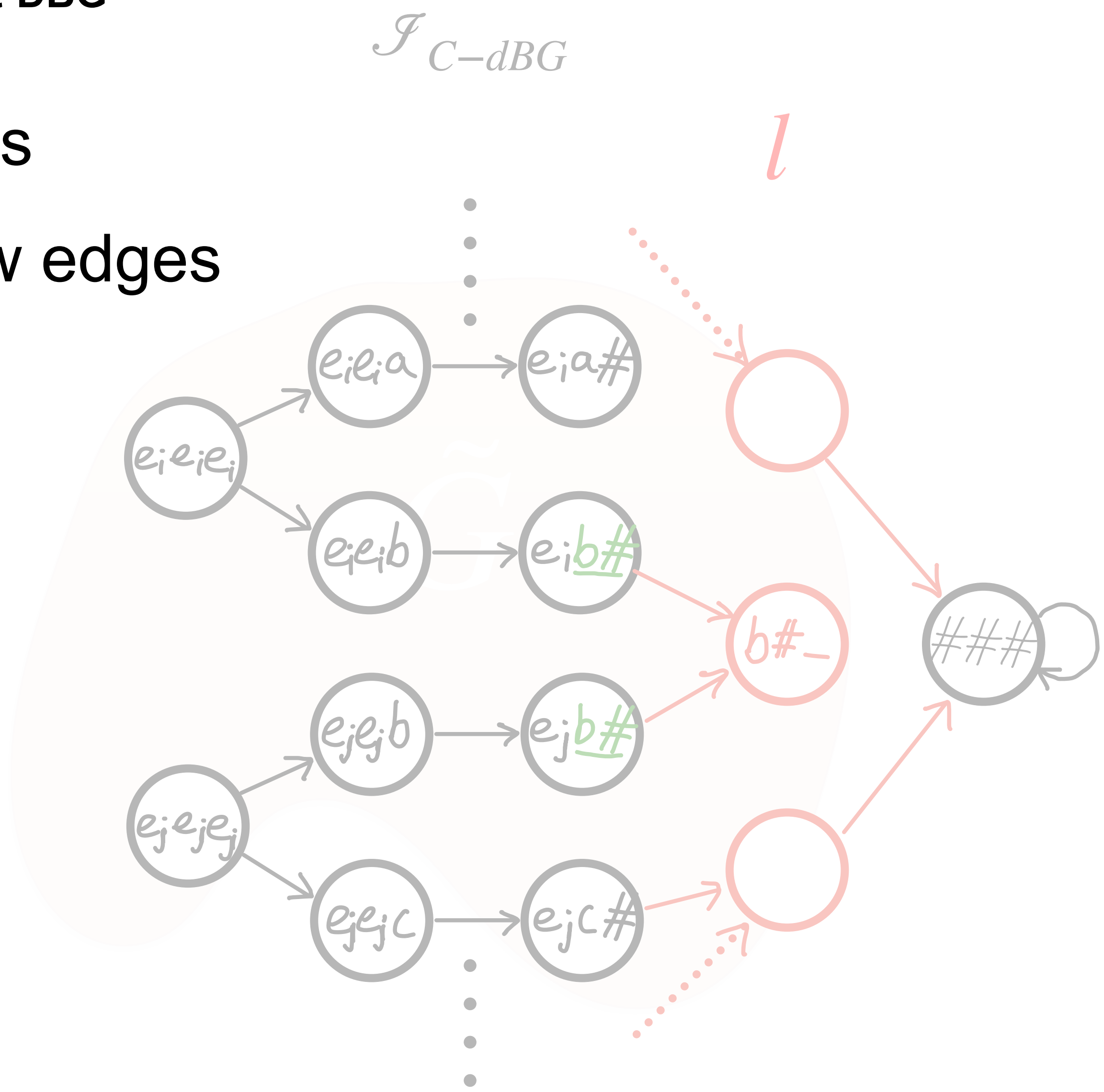
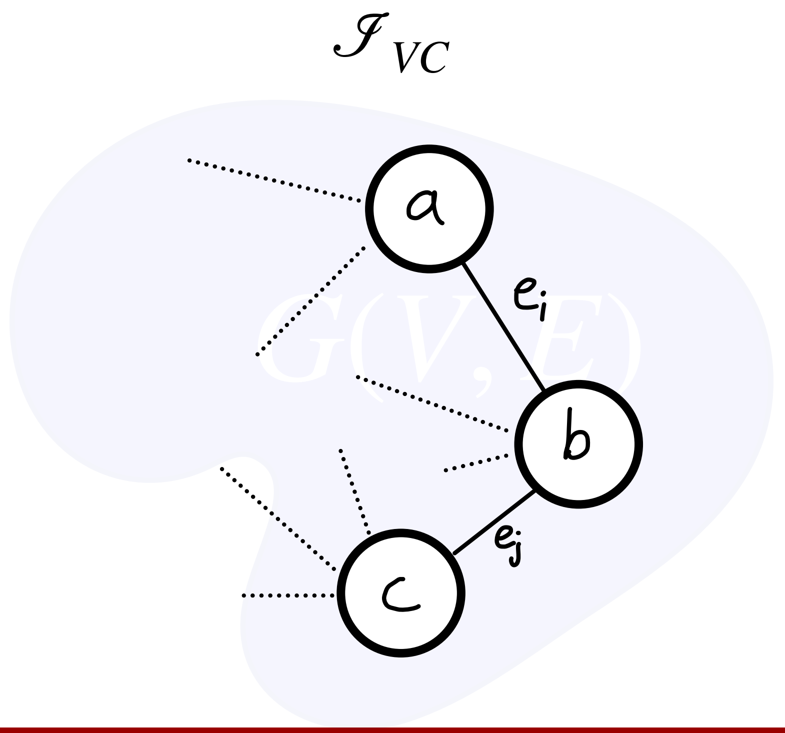


Hardness of Connect-DBG

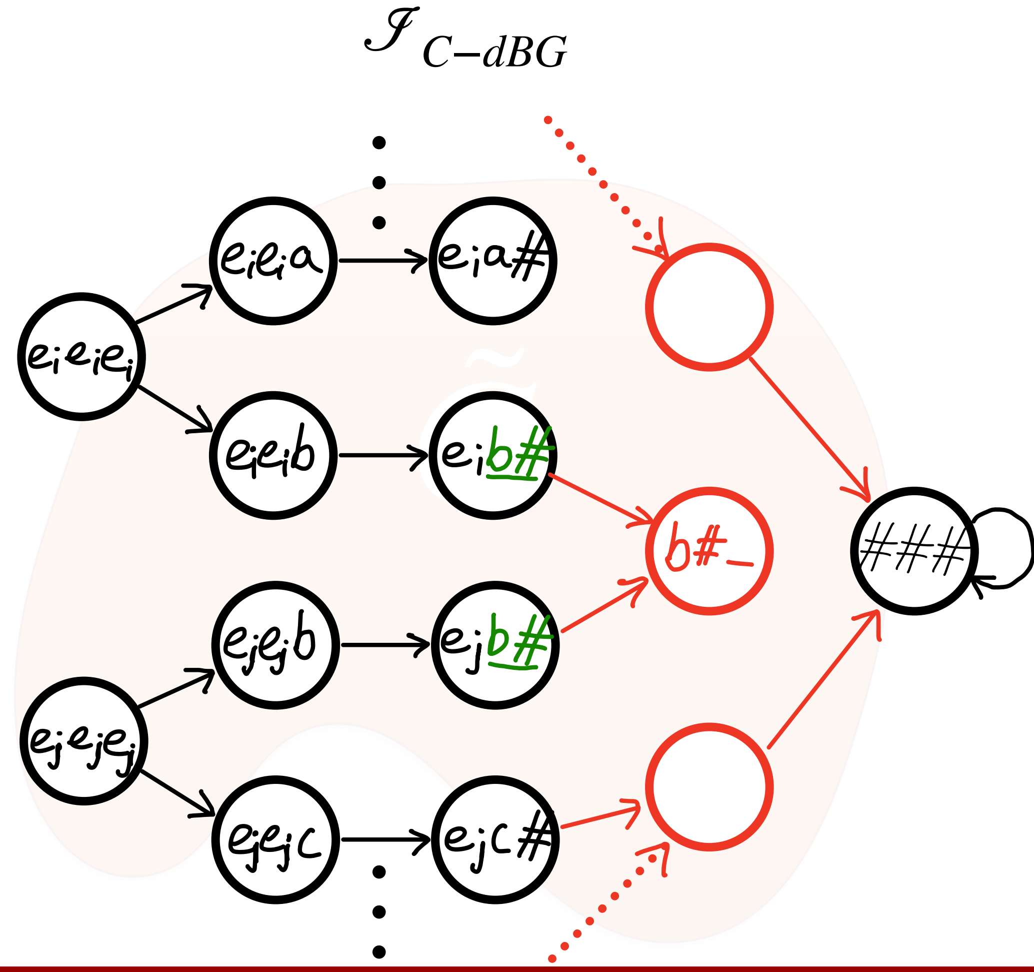
- l new vertices
- $|E| + l$ new edges



For every new vertex:



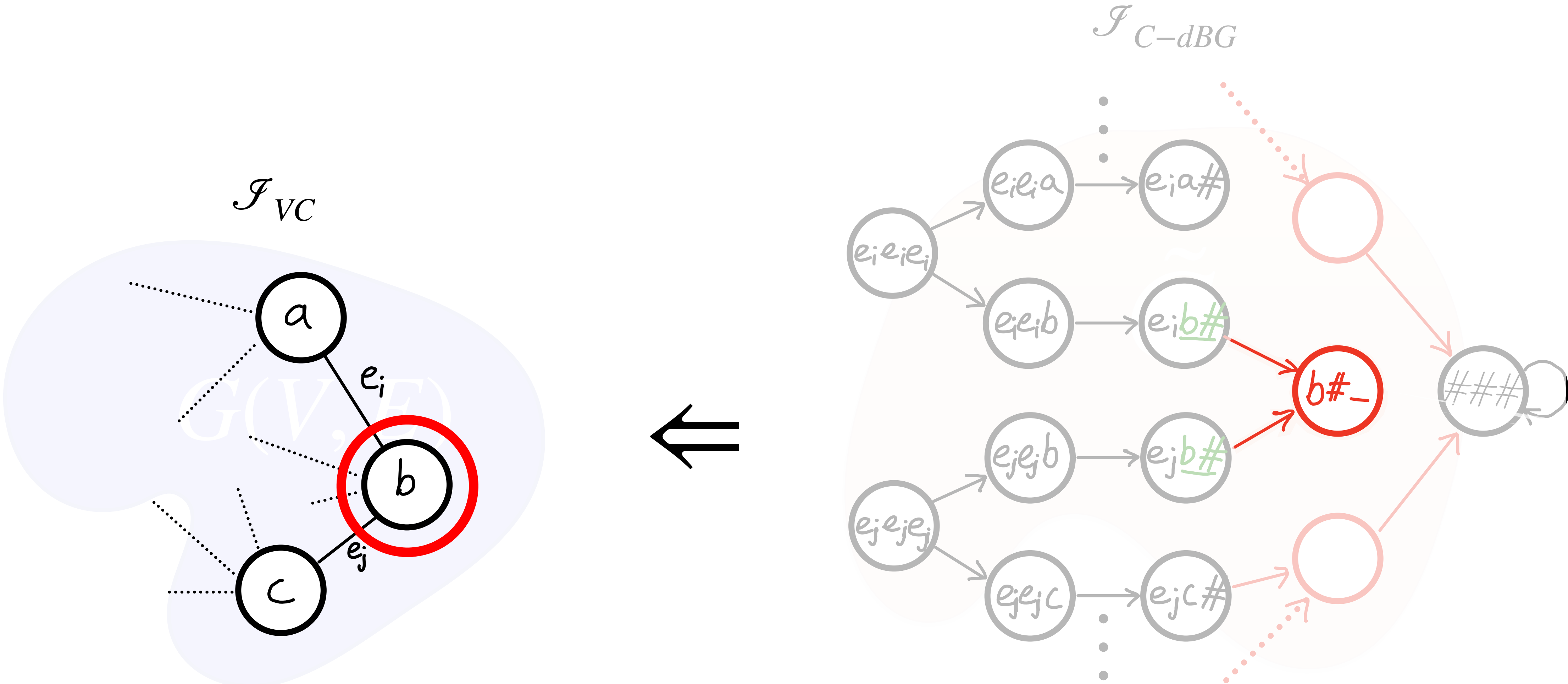
$\Leftarrow$



Hardness of Connect-DBG

For every new vertex:

- If 2 adjacent edge-gadgets → choose corresponding vertex



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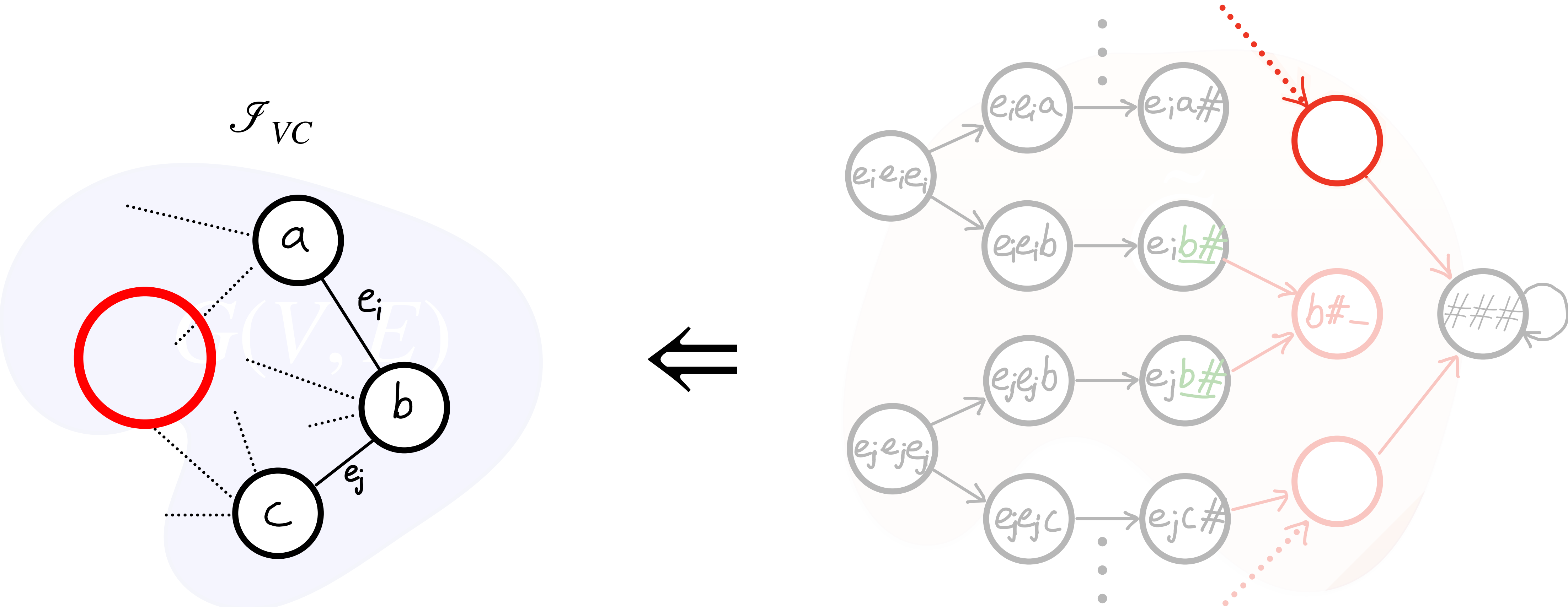
$\mathcal{I}_{VC}$ $\Leftarrow$ $\mathcal{I}_{C-dBG}$



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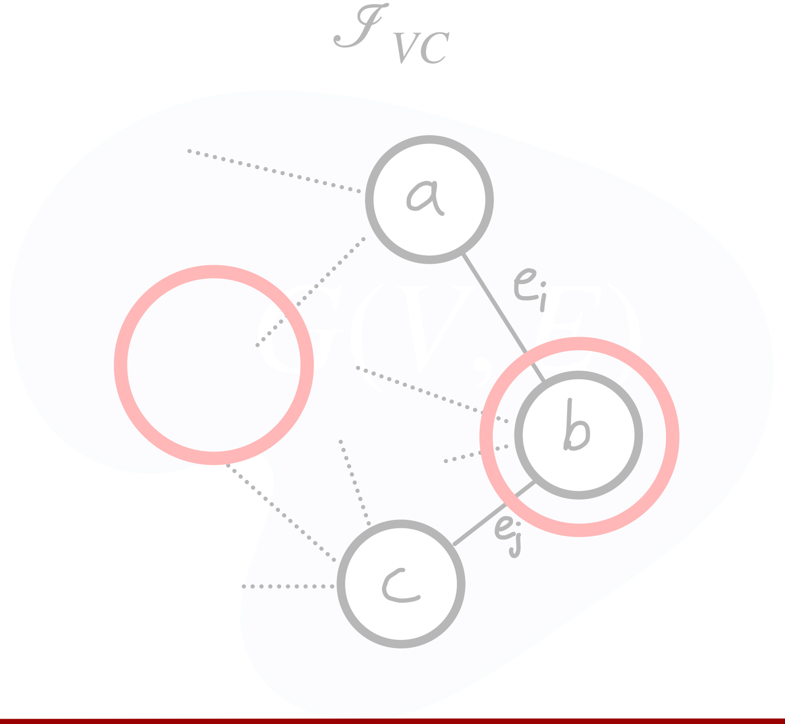
Solution to $\mathscr{I}_{VC}$ of size l



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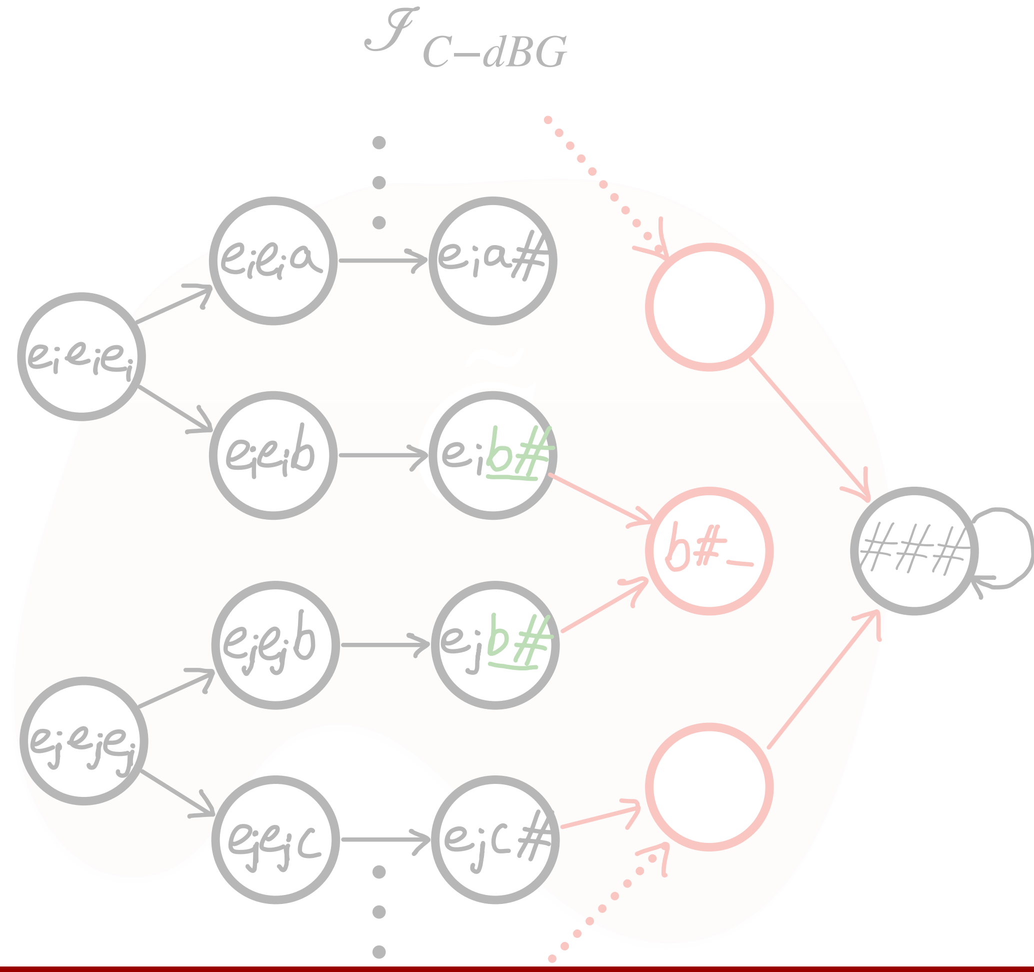

Hardness of Connect-DBG

$$OPT(\mathcal{I}_{VC}) = l \quad \Leftrightarrow \quad OPT(\mathcal{I}_{C-dBG}) = |E| + l$$

Approximation of CONNECT-DBG

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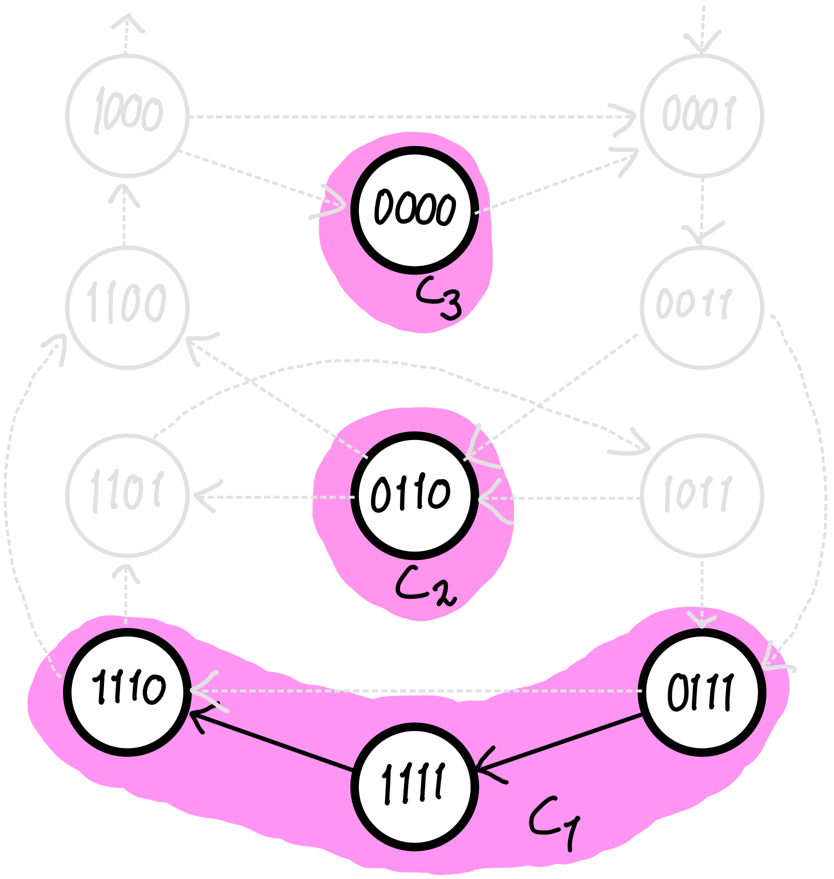
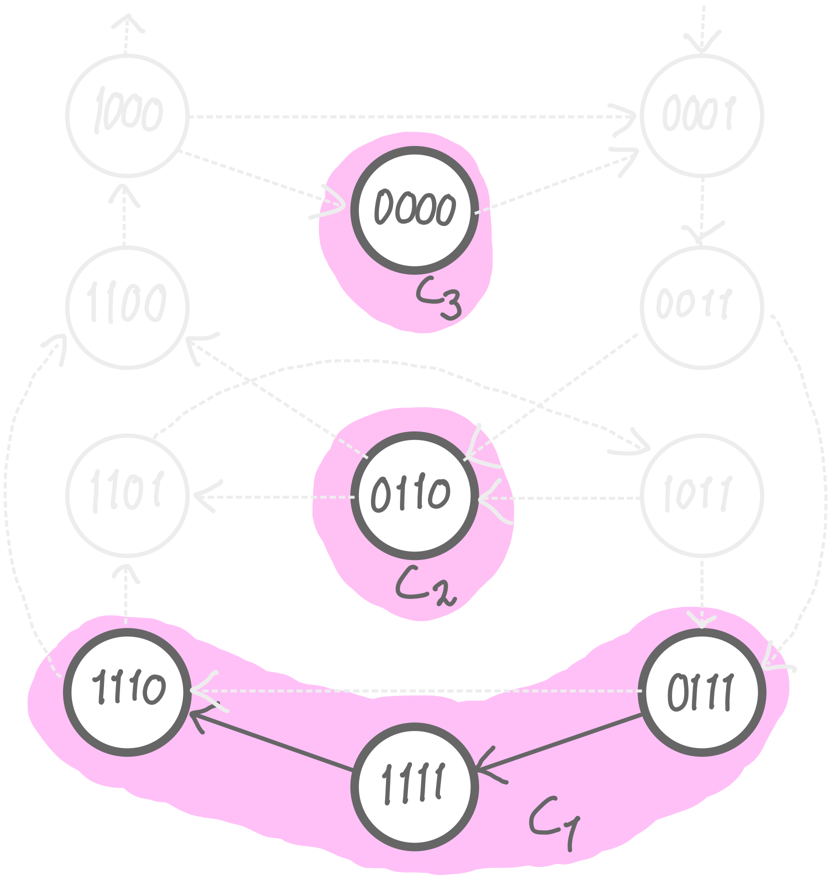


Approximation of Connect-DBG

- Collapse each connected component into a supernode in the complete dBG



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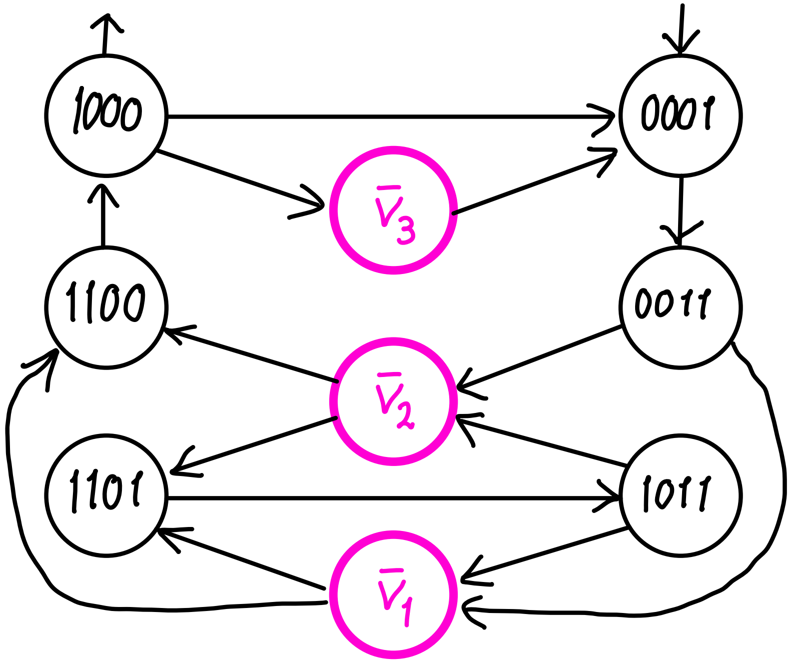
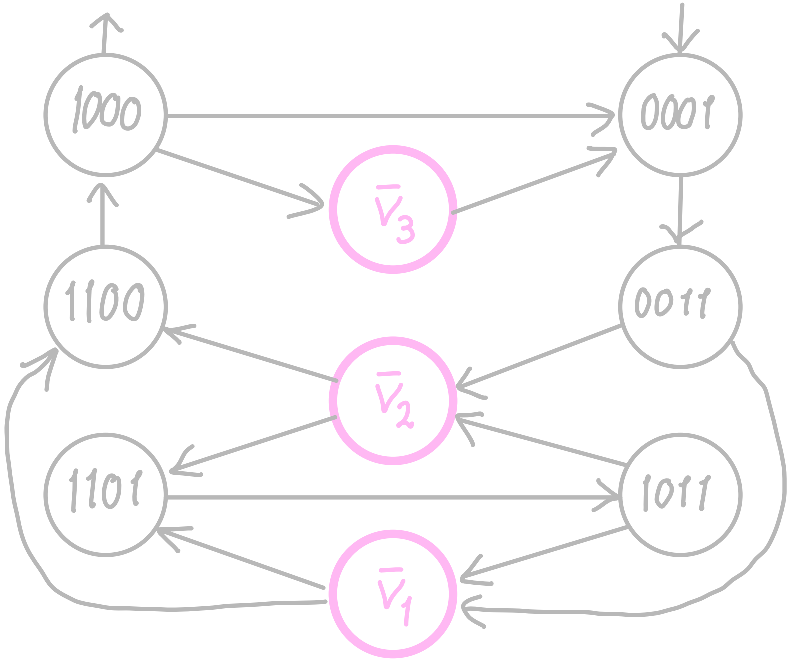


Approximation of Connect-DBG

- Construct the metric closure



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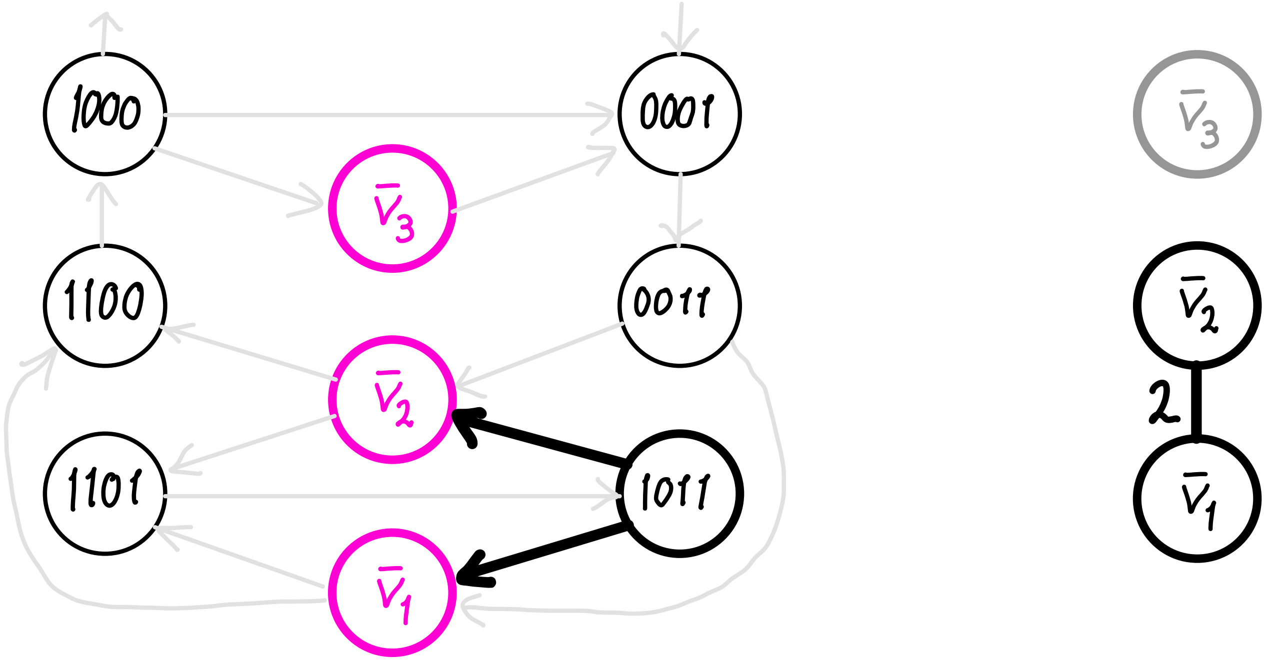

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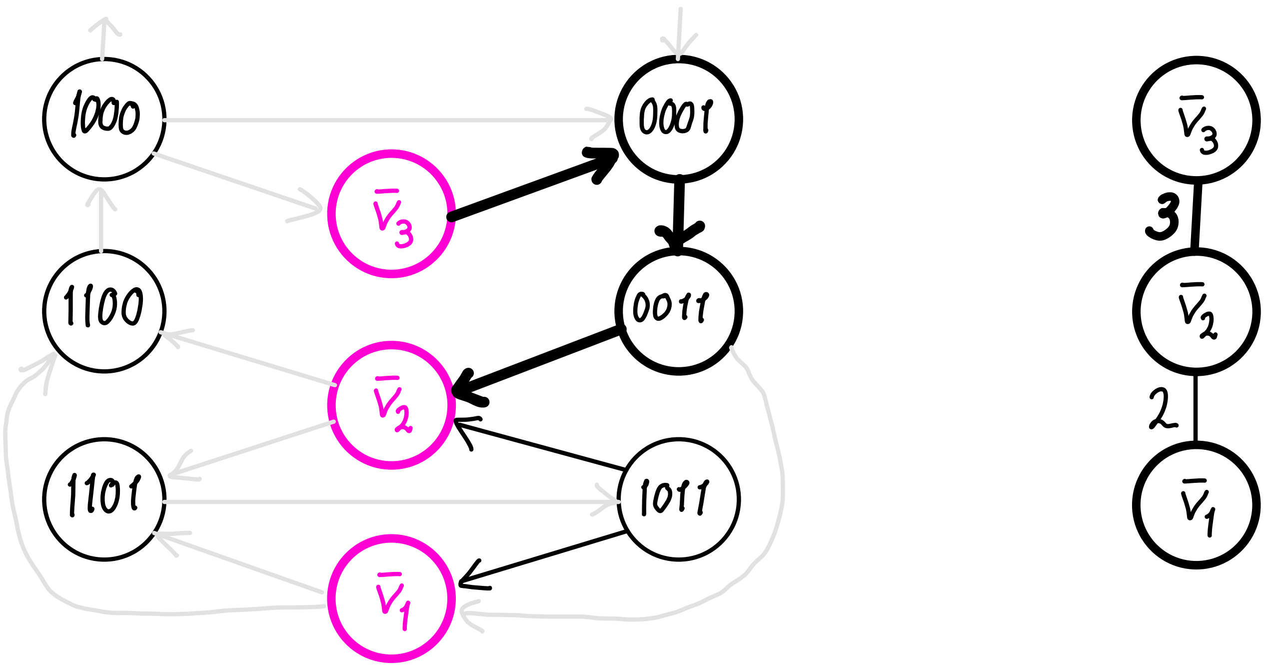

Approximation of Connect-DBG

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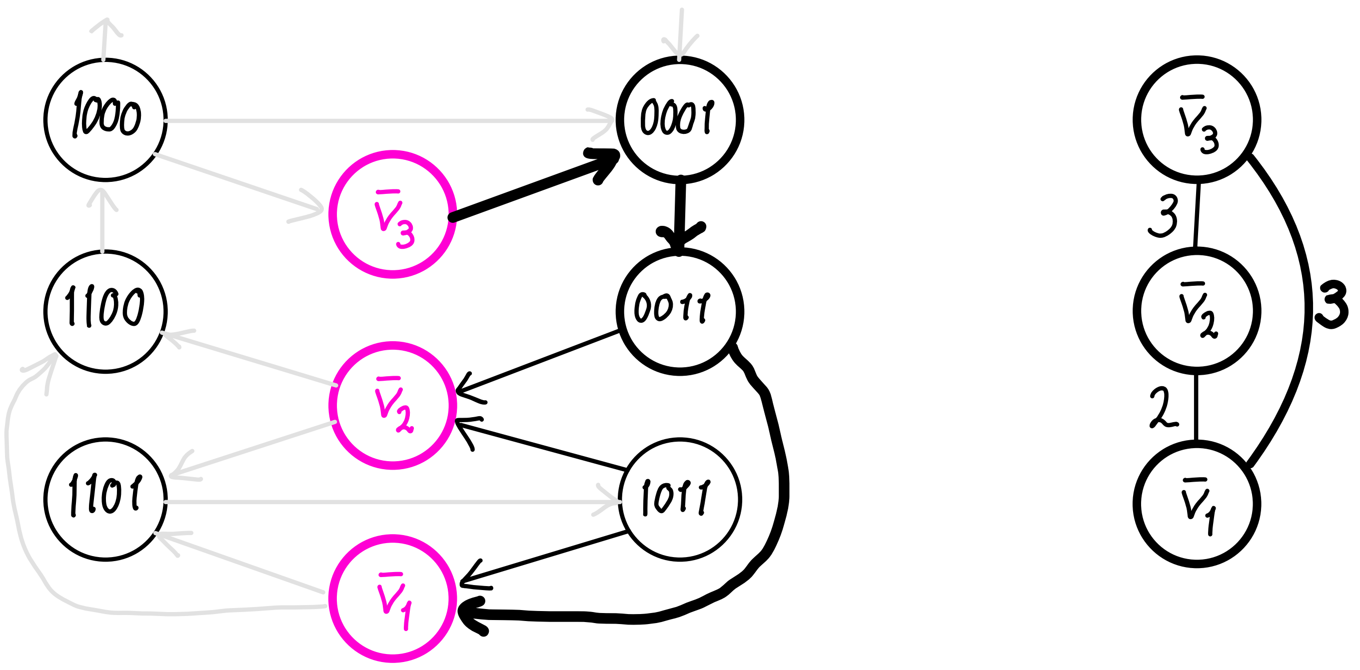
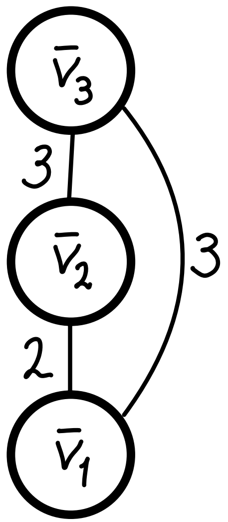


Approximation of Connect-DBG

- Use 2-approximation for metric closure of Steiner Tree Problem by Kou et al. (1981)[1]



[1] Lawrence T. Kou, George Markowsky, and Leonard Herman. A fast algorithm for Steiner trees. *Acta Informatica,* 15:141-145, 1981. doi:10.1007/BF00288961

Improvement of CONNECT-DBG-P

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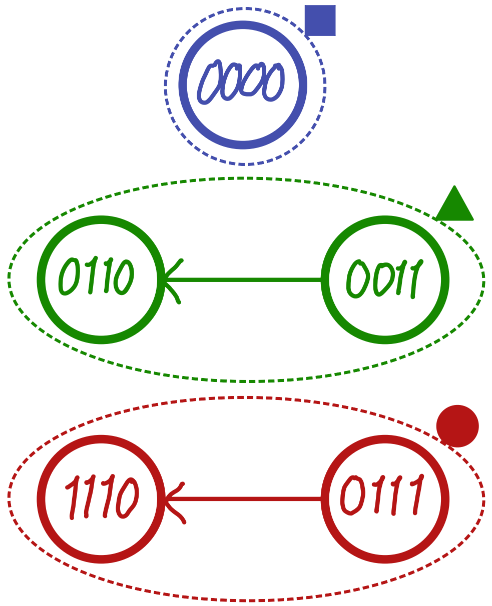

- Aho-Corasick (AC) Machine (KMP generalization)

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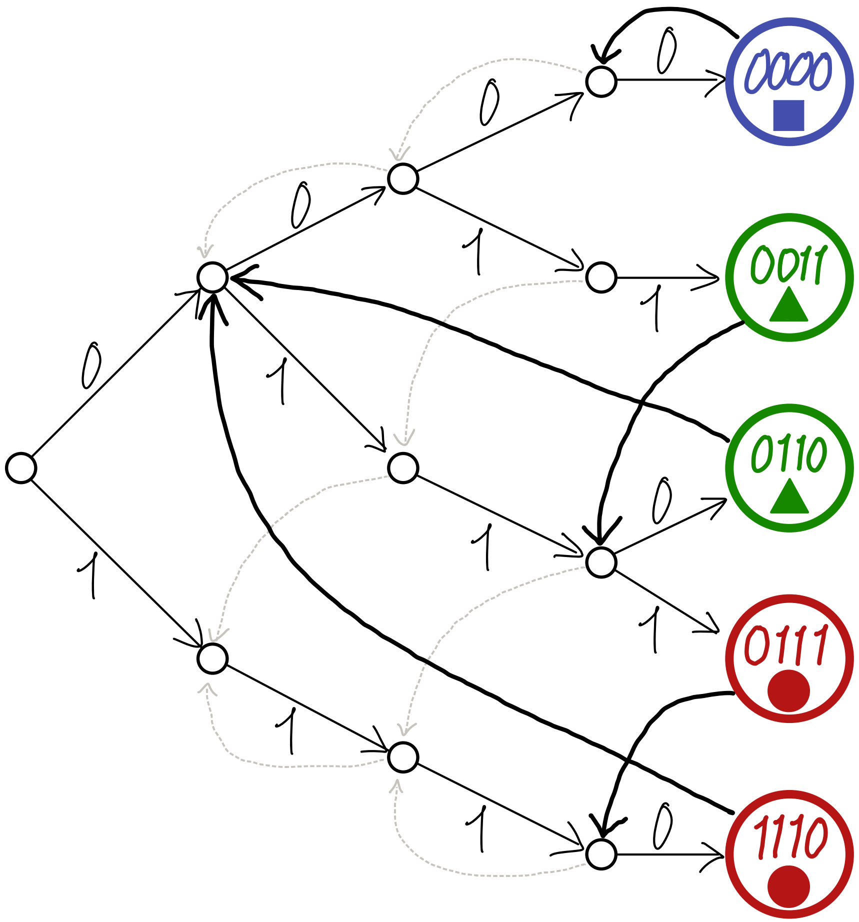
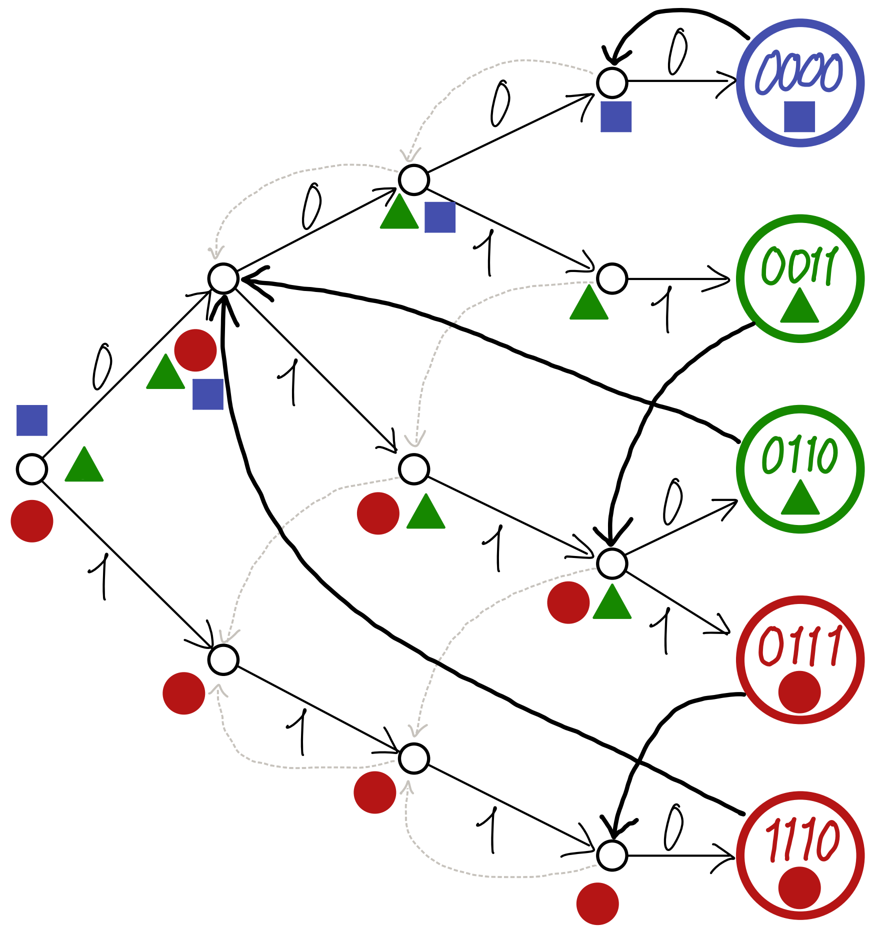


Improvement of Connect-DBG-P

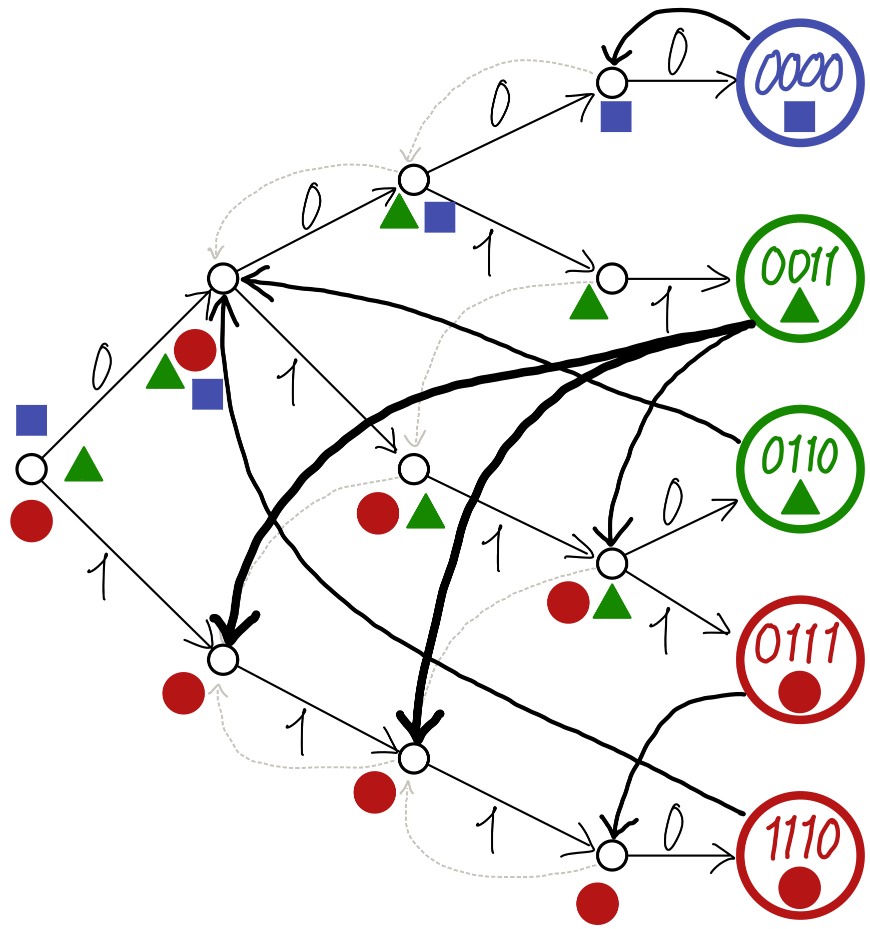
- Aho-Corasick (AC) Machine (KMP generalization)

- Add colors



Improvement of Connect-DBG-P

- Aho-Corasick (AC) Machine (KMP generalization)
- Add colors
- **Add backward edges**



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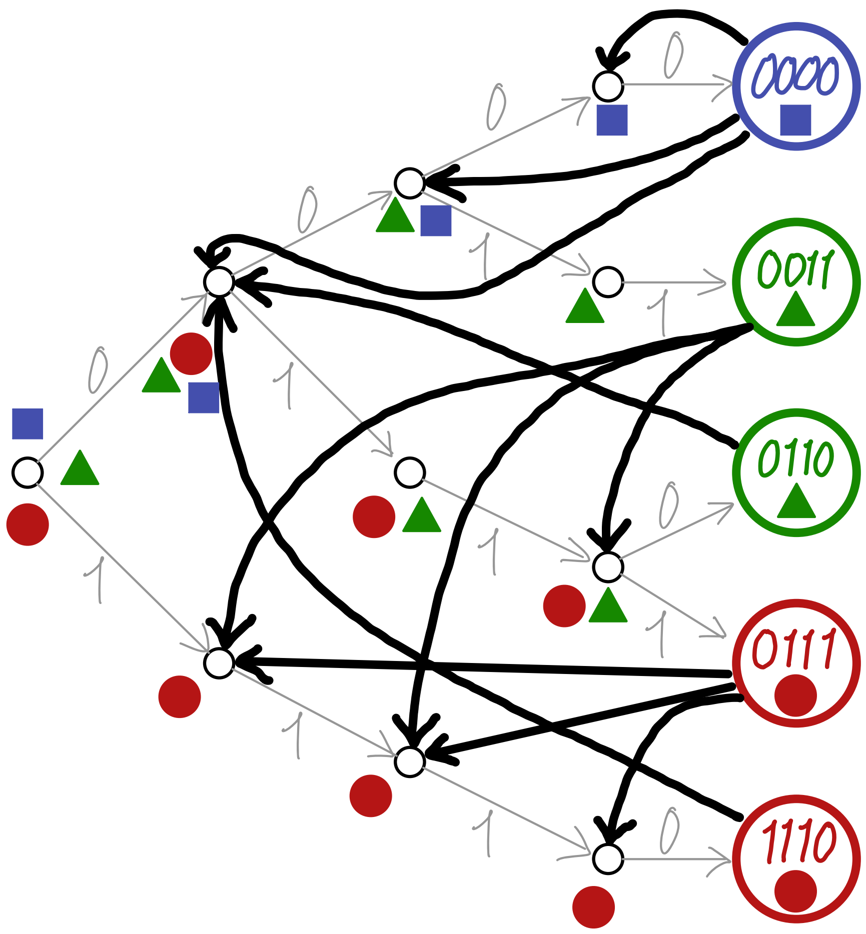

Improvement of Connect-DBG-P

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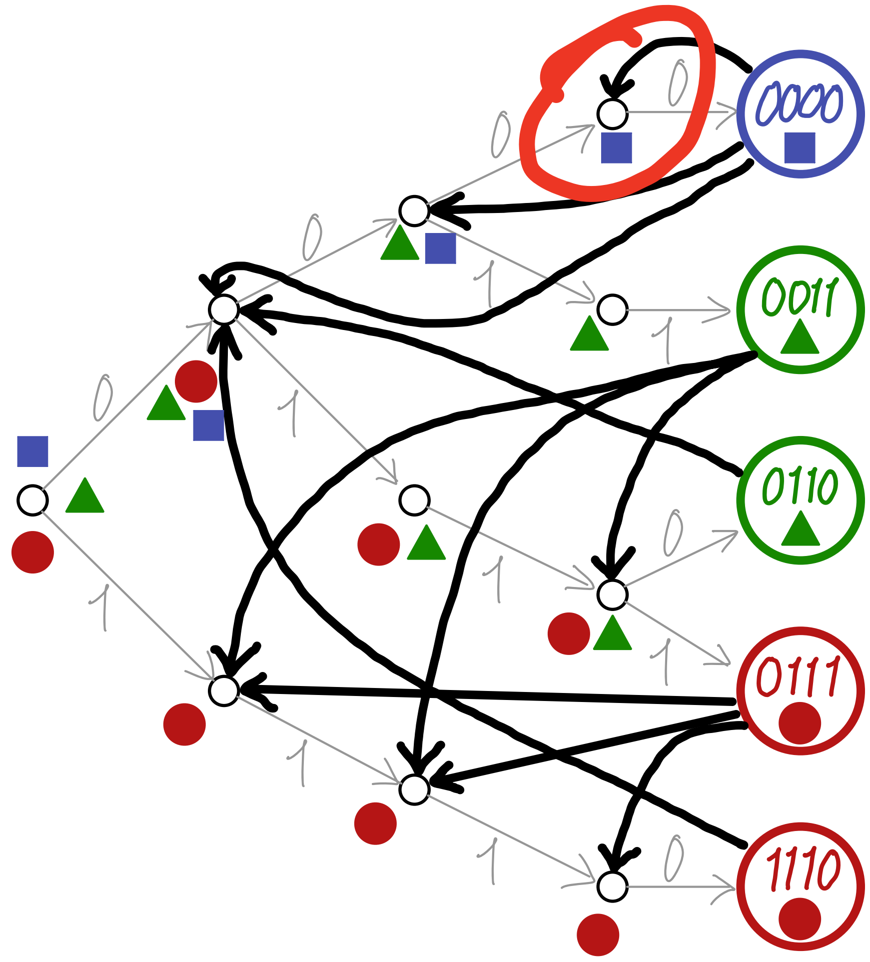
- Add colors
- Add backward edges

- **Reverse BFS from root:**
  - Compare backward edges to colors



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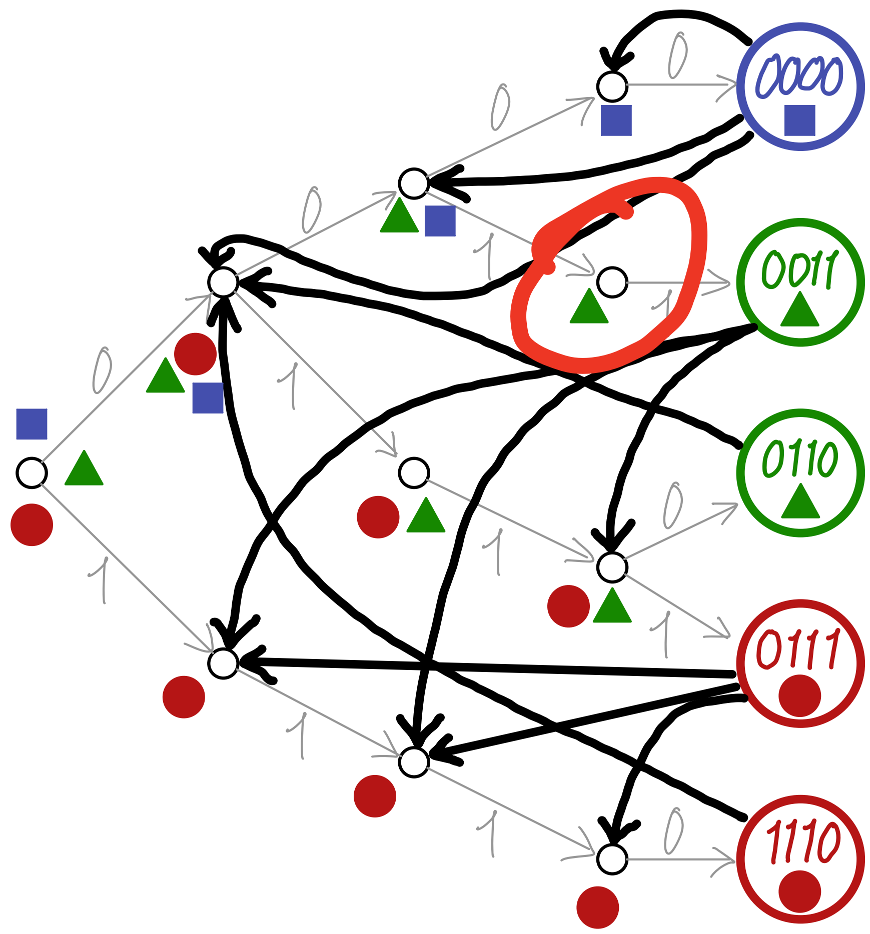

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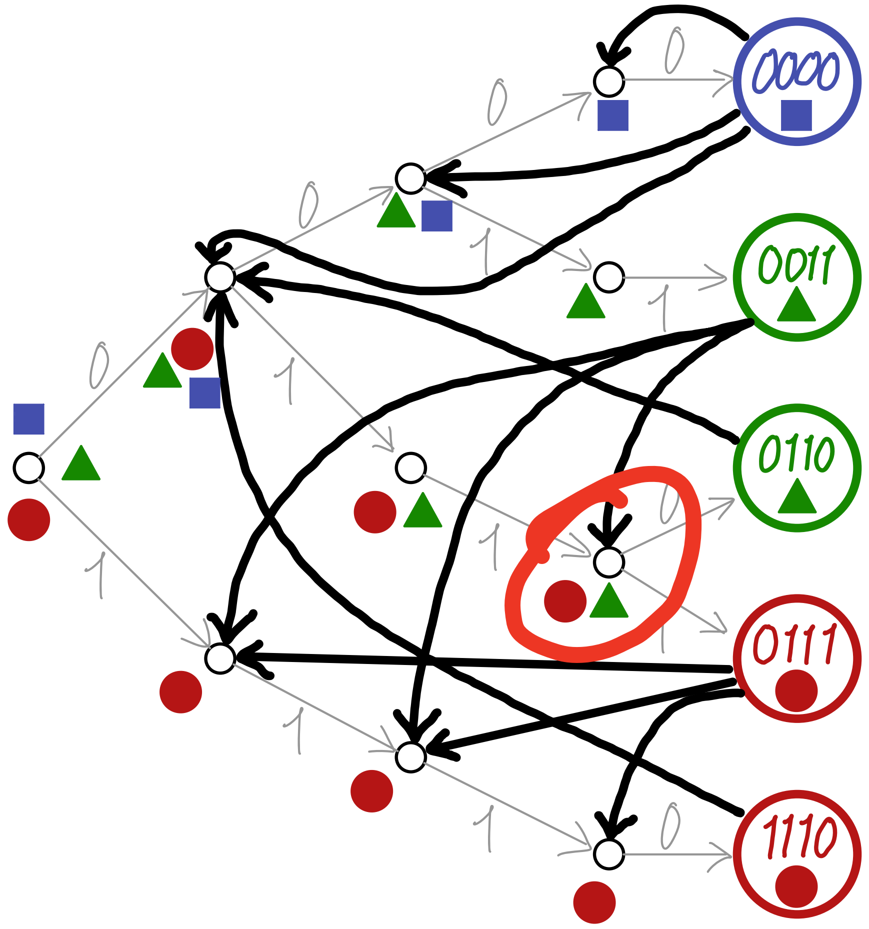

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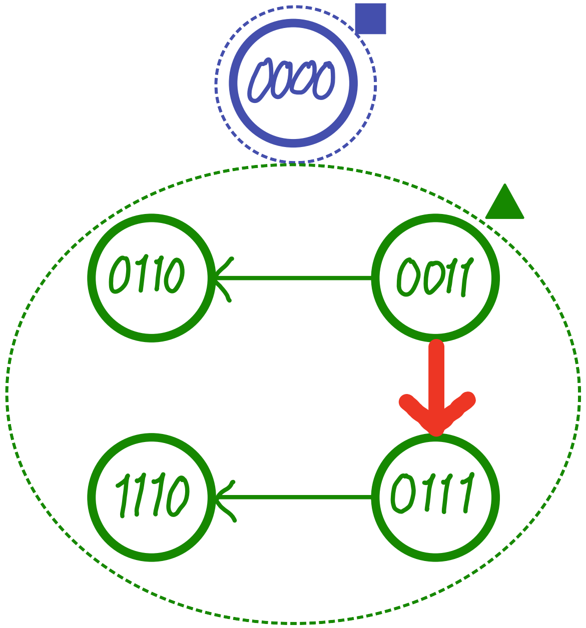

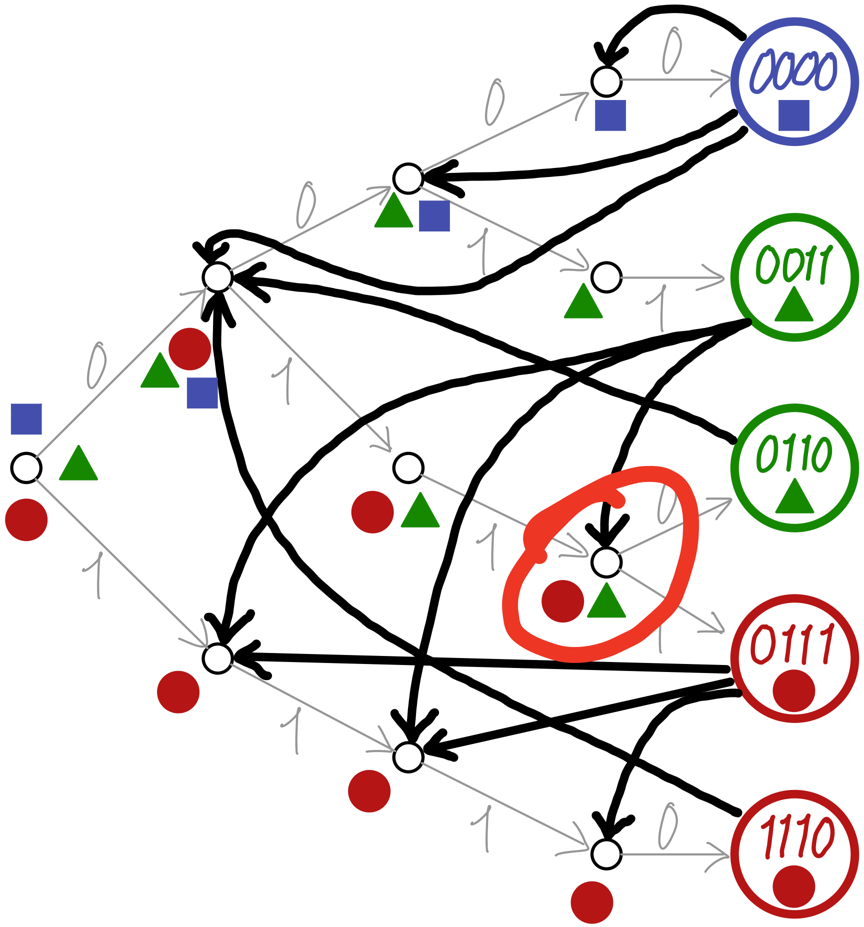

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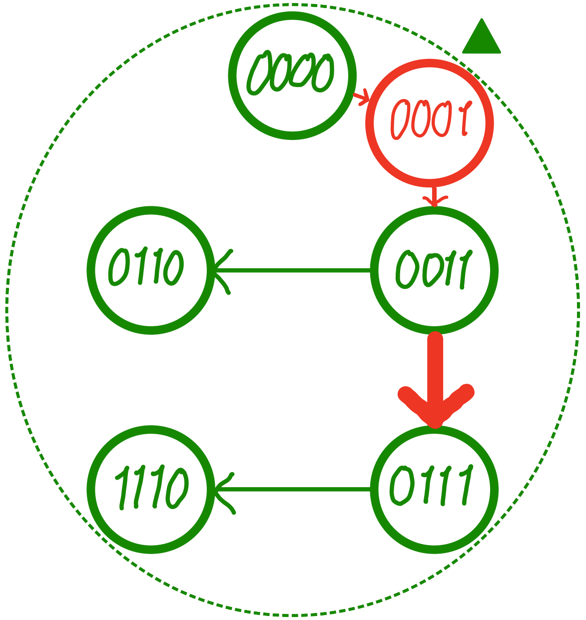

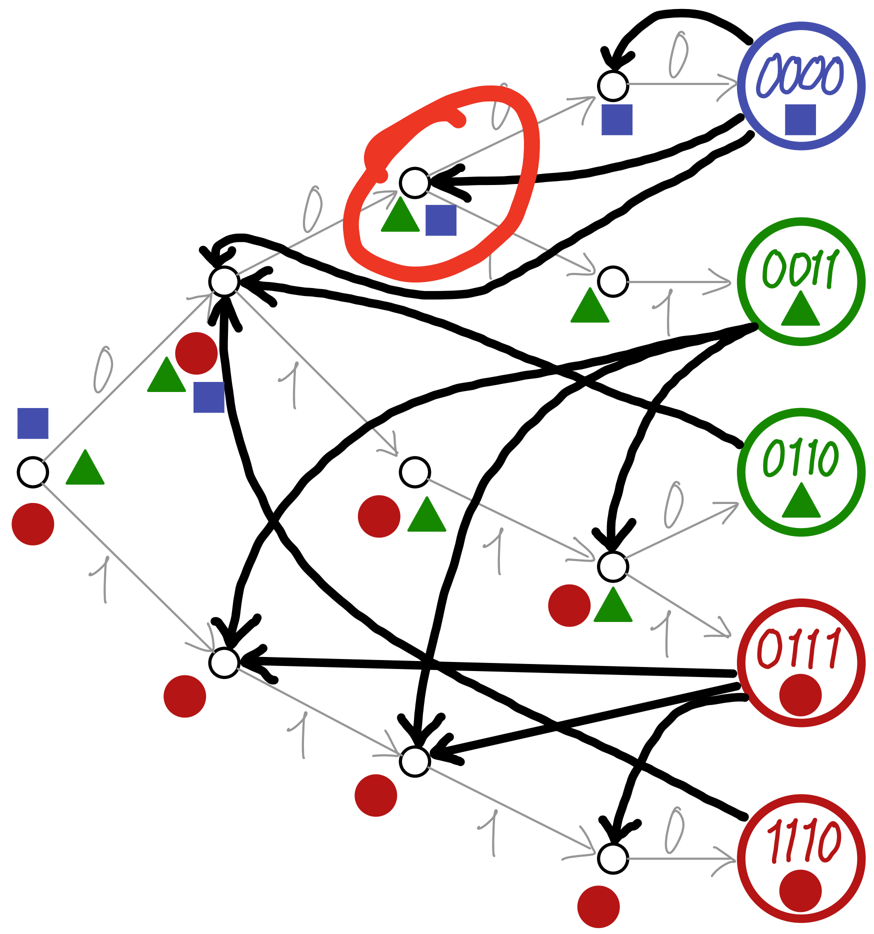

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$$\mathcal{O}(k|V|\alpha(|V|) + |E|) \ \text{time}$$

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$\alpha(\cdot)$ is the inverse Ackermann function

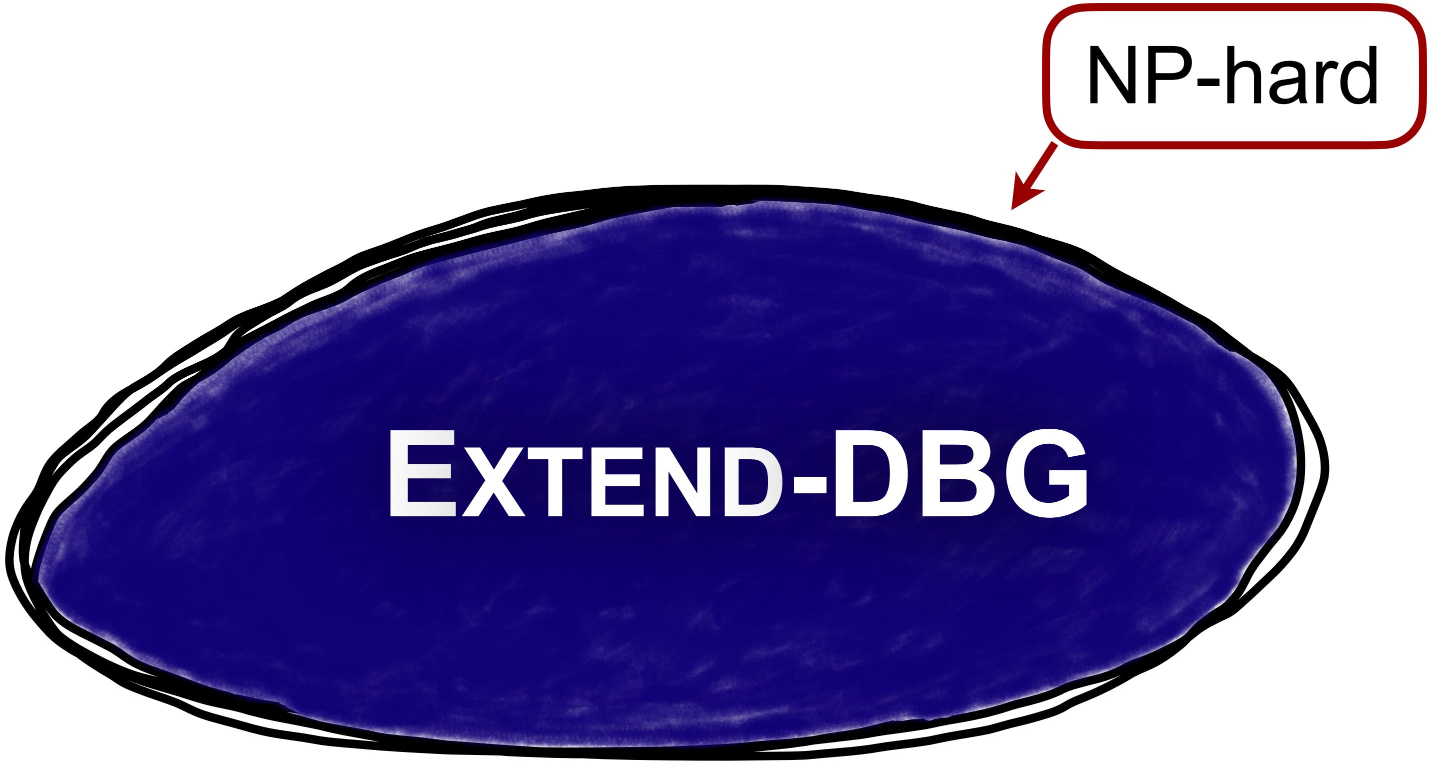
$\alpha(n)$ grows slower than $\log^* n$

Summary

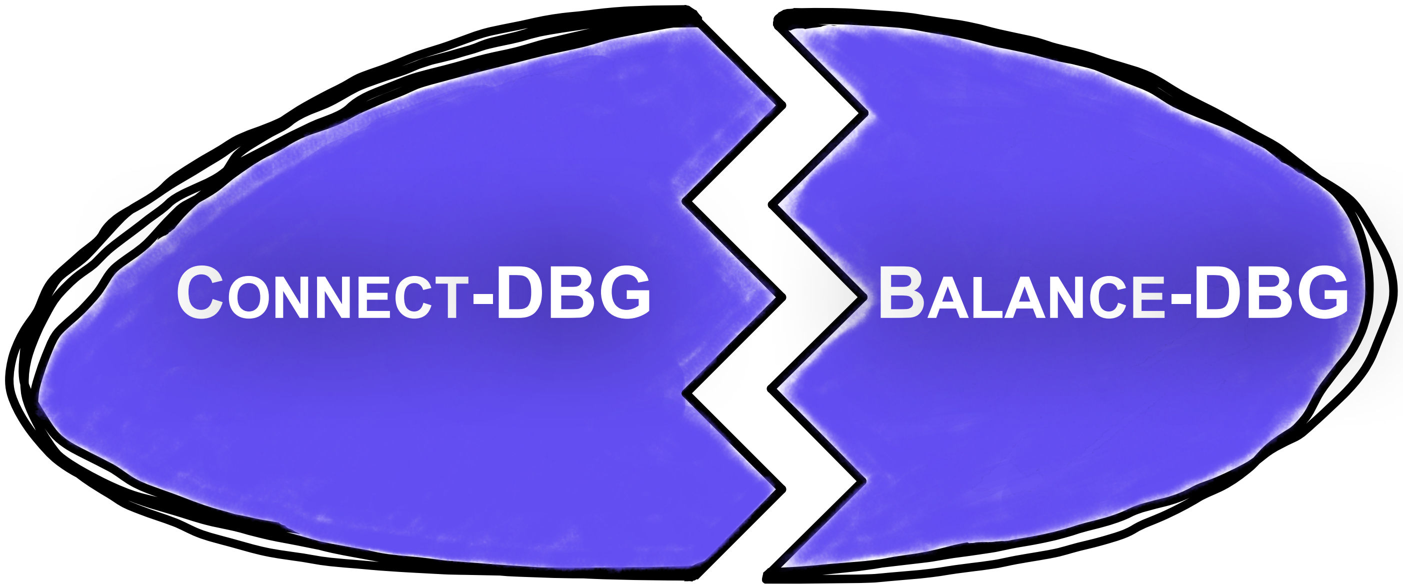
Summary

EXTEND-DBG

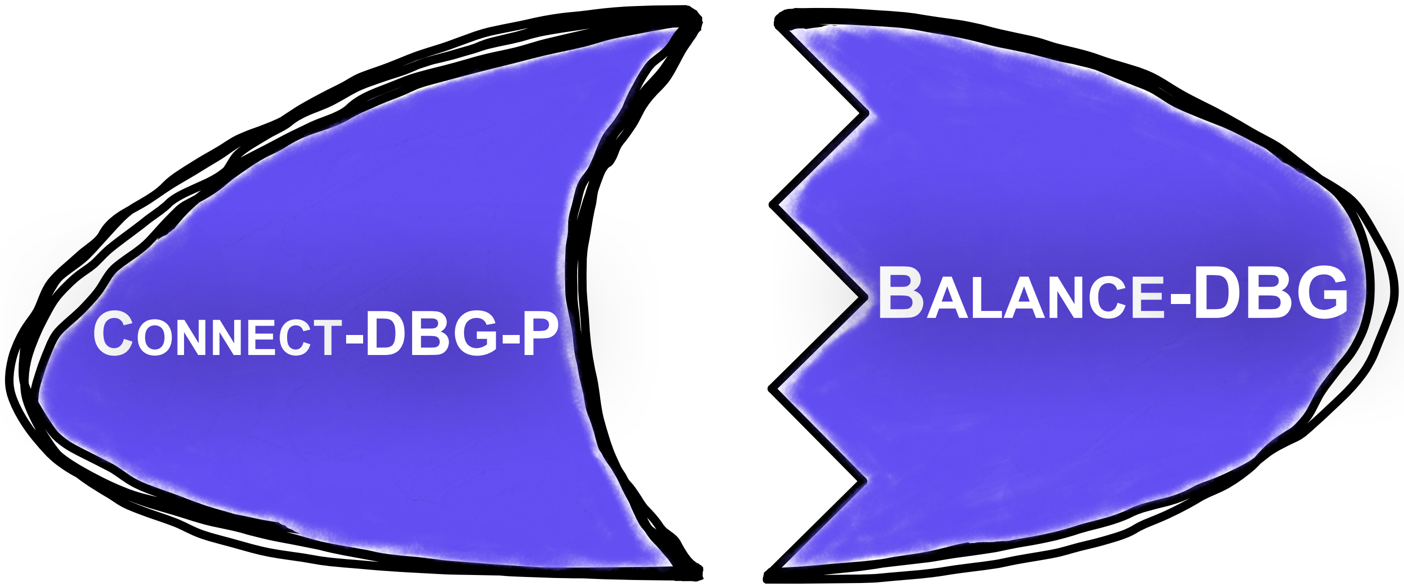
Summary



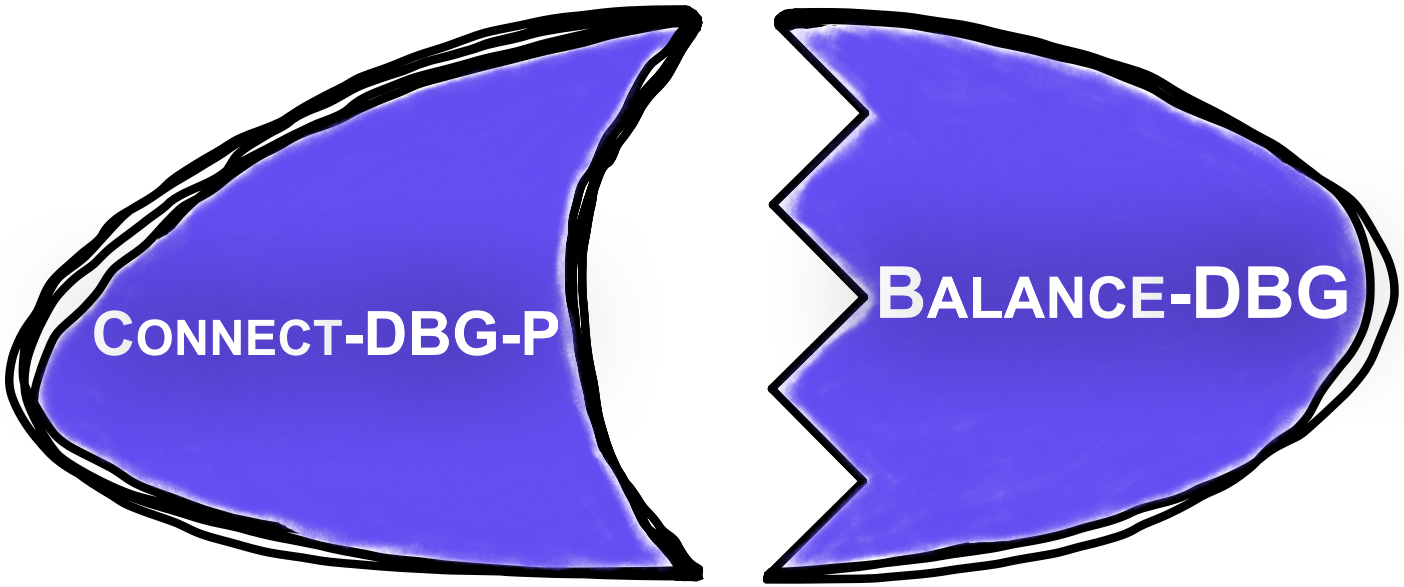
Summary



Summary



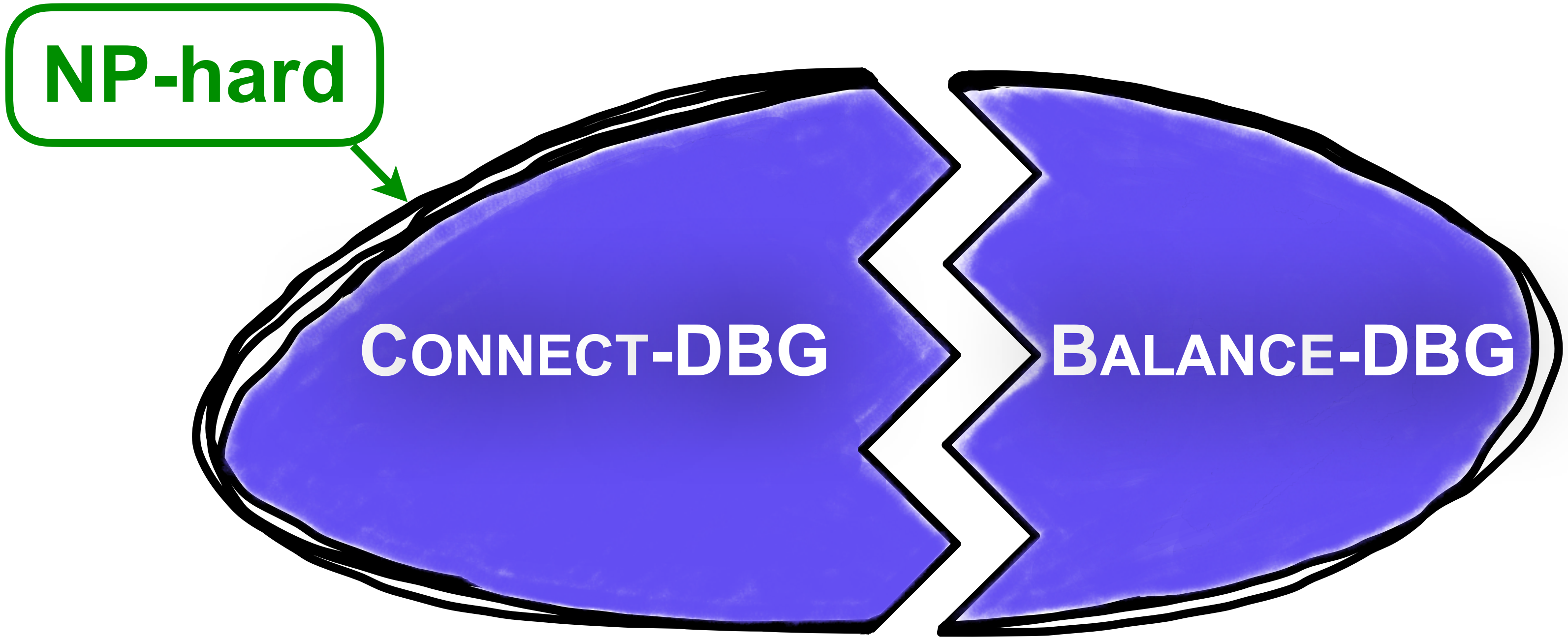
Summary



$\mathcal{O}(k|V|\alpha(|V|) + |E|)$ time

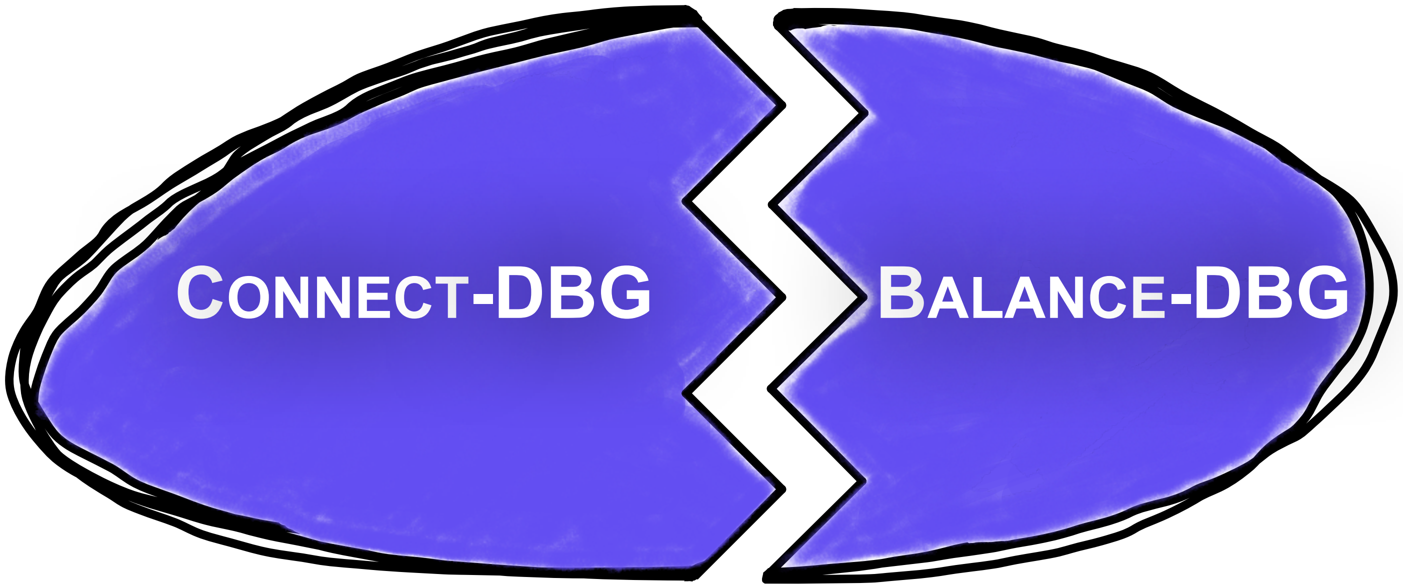
$\mathcal{O}(k|V| + |E| + |A|)$ time

Summary



$\mathcal{O}(k|V| + |E| + |A|)$ time

Summary



2-approximation $\mathcal{O}(k|V| + |E| + |A|)$ time

Summary

Extend-DBG

Approximation?